Divergence of Opinion and the Cross Section of Stock Returns

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Abstract

In this paper we generalize Pitts and Tauchen’s (1983) well-known Mixture of Distribution Hypothesis (MDH), which links asset volume and volatility in a way that derives a proxy for divergence of opinion among all individual investors. This new measure has several advantages over the existing proxies such as dispersion in analysts’ earnings forecasts and turnover. We then use this measure of divergence of opinion in an empirical asset pricing analysis. In particular, we incorporate the crucial role of divergence of opinion in the determination of cross-sectional asset returns, establishing that when divergence of opinion is high, stock prices tend to be biased upwardly, resulting in lower future returns. These effects are especially pronounced for small, low-book-to-market, and high-momentum stocks, which are more difficult and costly to short sell. Hence the evidence for these stocks support Miller’s (1977) view that, given short-sale constraints, observed prices overweight optimistic valuations. The predictions of recent theoretical work, such as Hong and Stein (2003), are valid only for stocks that are easier to short sell.

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Introduction

Does divergence of opinion play an important role in determining asset prices and asset returns? In a seminal paper, Miller (1977) argues that divergence of opinion does matter for asset pricing when short sales constraints are binding, so that only optimistic investors’ valuations are reflected in the stock price. In addition, Miller predicts that stocks with greater divergence of opinion should have higher current prices but lower future returns. In contrast, Hong and Stein (2003) argue that the stock prices aggregate all investors’ valuations in an unbiased way and are thus not affected by divergence of opinion. Their conclusion is, however, based on the strong assumption that timely and frictionless arbitrage is available in the economy.¹

We conjecture that there is some truth to both sides of this argument. On the one hand, small-capitalization, low-book-to-market (glamour) and high-momentum stocks tend to be more difficult and more costly to short sell. Divergence of opinion will thus have a significant effect on these stocks as predicted by Miller (1977). On the other hand, it is much easier to short sell large, high-book-to-market (value), and low-momentum stocks, therefore divergence of opinion does not play a significant role in the returns of these stocks, as predicted by the Hong-Stein model. In this paper, we test the above hypothesis and examine the effect of divergence of opinion on cross-sectional stock returns by sorting stocks according to their characteristics such as size, book-to-market and momentum.

We first derive a new proxy for divergence of opinion among all individual investors, based on Pitts and Tauchen’ (1983) structural model for volume and volatility. Prior empirical studies of the divergence effect on equity returns have produced mixed results, partly because the literature fails to identify a reliable proxy for divergence of opinion among all individual investors. Pitts and Tauchen’ (1983) model has been used

¹Some theoretical work, e.g. Varian (1985) and Merton (1987), indicates that divergence of opinion has a positive relationship with future asset returns.
in many other applications, however it has never been used in the divergence of opinion area.\(^2\) There is one variance term in the model, which represents the degree to which people disagree with each other, can be a very good proxy for divergence of opinion after some modifications. This variance term directly isolates the information of divergence of opinion from the relation of volume and volatility. However, it is only a constant parameter in their model. We generalize this variance term into a time-varying process so that it can explain the time-varying process of volume and volatility. This new proxy for divergence of opinion (DIV) is not only strongly and positively related to the existing proxies for divergence of opinion, such as analysts’ earnings forecasts and turnover, but it also provides a closer link to the theory advocated in Miller (1977) and Hong and Stein (2003). Our measure captures the divergence of opinion among all individual investors instead of merely among financial analysts, who may issue biased opinions and herd among themselves due to conflicts of interest and concerns for their job security.\(^3\) Moreover, some investors do not follow analysts’ forecasts and even those who do rely on analysts’ recommendations may have their own way of aggregating this information.\(^4\) This makes dispersion in analysts’ earnings forecasts a noisy and imprecise proxy for divergence of opinion for all investors. Our proxy is also a cleaner measure of individual investors’ divergence of opinion than asset turnover, because the latter is also used as a proxy both for asset liquidity and for information-induced price adjustment speed.\(^5\) Finally, our measure goes back to January 1970, enabling us to investigate the effect of divergence of opinion on equity returns over a longer sample

\(^2\)It has been used to analyze the relation between volatility and volume. See Anderson (1996), Liesenfeld (2001).

\(^3\)Michaely and Womack (1999), Bradshaw, Richardson, and Sloan (2003), and Jegadessh, Kim, Krische, and Lee (2004) indicate that the incentive structure of analysts induces them to give positive reports in order to win investment bank deals. Scherbina (2004) documents that analysts’ earnings forecasts bias tends to be larger, the bigger the disagreements about the firms earnings among analysts.

\(^4\)Begnoli, Mark, Beneish and Watts (1999) document that investors have been following online unofficial “whisper” forecasts in recent years.

period.

Using this new measure of divergence of individual investors’ opinions, we test our hypothesis and investigate how divergence of opinion affects equity returns when stocks are sorted according to characteristics, such as size, book-to-market ratio, and momentum. The rationale is derived from the wide documentation that small, low-book-to-market and high-momentum stocks are more difficult to short sell. For example, it is argued by Merton (1987) and Grossman and Miller (1988) that small stocks might have less arbitrage capacity. It is thus more difficult to short sell and find trading options for these small stocks. Geczy et al. (2002) find that loan fees are five times higher for low-book-to-market (glamour) stocks than high-book-to-market (value) stocks. This implies that it is more costly to short sell glamour stocks than value stocks. Finally, D’Avolio (2002) finds that lenders prefer large stocks with high book-to-market values and low momentum. The empirical results support our hypothesis and contribute to resolving the debate on the effects of divergence opinion on future stock returns. We find that stocks with high divergence of opinion tend to be overpriced and that the average return differences between low-divergence and high-divergence stocks are significantly positive, with the highest decile of DIV outperforming the lowest decile of DIV by 8 percent annually. Consistent with our hypothesis, these effects of divergence of opinion on equity returns are concentrated on small, glamorous, and high-momentum stocks, which are more difficult and costly to short. When we sort all stocks into five equal groups based on the stocks’ market capitalization, we find that the low divergence of opinion stocks outperform the high divergence of opinion stocks by over 17 percent, 15 percent and 6 percent per year in the smallest three groups, and 17 percent, 14 percent and 7 percent respectively after the risk adjustment. However, the return difference between the low and the high divergence of opinion stocks in the two large-size groups either has the wrong sign or is statistically insignificant. Among small stocks, it is the glamorous growth stocks and the high-momentum stocks that are most strongly af-
fected by divergence of opinion in terms of both economic and statistical significance: the low divergence of opinion small growth (high-momentum) stocks outperform their high divergence of opinion counterparts by almost 23 percent (26 percent) per year after the risk adjustment with a t-ratio of 11.40 (12.02). Even for mid size stocks, this effect is still strong with low divergence of opinion growth (high-momentum) stocks outperforming their high divergence of opinion counterparts by 7 percent (12 percent).

In the robustness analysis, we carry out a Fama-MacBeth cross-sectional regression, where the cross-sectional expected returns of all individual stocks are regressed on divergence of opinion measure (DIV), market beta, size, book-to-market, and past returns. Interestingly, neither the market beta nor size has any significant explanatory power for cross-sectional returns. The divergence of opinion variable, DIV, has a coefficient of -0.08 and is highly statistically significant even after we control for market risk, firm size and book-to-market ratio. This is consistent with the hypothesis that stocks with high divergence of opinion have lower future returns. To examine whether DIV has additional explanatory power beyond the existing measures, divergence of analysts’ forecasts and asset turnover, we repeat the regression by including these two variables as additional regressors. While the coefficient for DIV remains negative and significant at a 10 percent level, neither the divergence of analysts’ forecasts nor asset turnover enters the regression significantly.

In addition to the above robustness analysis, we carry out subperiod analysis for three sample periods: 1970-1982, 1983-1992 and 1993-2003. The results show that divergence of opinion has effects on stock returns for all stocks during the first and second subperiods, and, further, that the effects are significant during all three subperiods for small stocks. The magnitude of the return difference, however, declines over time. For example, the low divergence of opinion small stocks outperformed their high divergence of opinion counterparts by as much as 1.85 percent per month on average during the first sample period, but the rate declined to 1.40 percent and 1.01 percent per
month for the second and third period, respectively. The decline in magnitude suggests that the effect of divergence of opinion on equity returns has decreased as the costs or difficulties in short selling stocks have gradually declined.

Our research relates to several empirical studies in this area. For example, Gragg and Malkiel (1982) report a positive relationship between future returns and dispersion of forecasts among a subset of analysts. Lee and Swaminathan (2000) find turnover has a negative relationship with future returns. Diether, Mallory, and Scherbina (DMS) (2002) find that high dispersion in analysts’ earnings forecasts is associated with low future returns, but they fail to provide clear evidence linking this negative dispersion-return relation to short-sale constraints. Compared to these existing empirical studies, our paper contributes to the literature in two ways. First, we have the advantage of using a more reliable and cleaner measure of the divergence of opinion among all individual investors. Second, we pay special attentions to the theoretical implications for different subsets of stocks by relating their characteristics to the assumptions of the models.

The paper comprises five sections. Section I provides the basic model of Pitts and Tauchen (1983) and further generalizes it into a model with time-varying divergence of opinion. Section II goes on to describe the data, provides statistics that show the relationship between DIV and the existing proxies, and analyze the cross-sectional distribution of divergence of opinion over time. The empirical analysis of the effect of divergence of opinion and equity returns is presented in Section III. Section IV shows various robustness tests and discusses the relationship of these results to the existing literature. Finally, Section V states the conclusion and proposes directions for future study.
I. Methodology

1.1. Basic Model of Pitts and Tauchen (1983)

The mixture of distribution hypothesis (MDH), proposed by Clark (1977) and further developed by Pitts and Tauchen (1983), states that there is a common information flow that directs both volume and volatility. In Pitts and Tauchen (1983), there are $J$ active traders and news arrivals cause $I$ equilibria within the day $t$. News arrives periodically within the day which causes the traders with diverse beliefs to revise their expectations. Each news arrival leads to a new round of trading until the market clears and a new equilibrium is reached.

$P^*_ij$ is the $j$th trader’s reservation price at the $i$th within-day equilibrium and $P_i$ is the market price at the $i$th within-day equilibrium. Let $Q_{ij}$ denote the desired position of the $j$ th trader at the time of $i$th within-day equilibrium. Then we have

$$Q_{ij} = \alpha [P^*_ij - P_i], \quad (j = 1, 2, ..., J), \quad (1)$$

where $\alpha > 0$ is constant. Market clearing condition $\frac{1}{J} \sum_{j=1}^{J} Q_{ij} = 0$ implies that the market price $P_i$ need to be expressed as

$$P_i = \frac{1}{J} \sum_{j=1}^{J} P^*_ij. \quad (2)$$

Equilibrium conditions give the price change and trading volume at the $i$th equilibrium:

$$\Delta P_i = \frac{1}{J} \sum_{j=1}^{J} \Delta P^*_ij, \quad (3)$$
\[ V_i = \frac{1}{2} \sum_{j=1}^{J} |Q_{ij} - Q_{i-1,j}| = \frac{\alpha}{2} \sum_{j=1}^{J} |\Delta P_{ij}^* - \Delta P_{i-1}|. \quad (4) \]

They assume a variance-components model as follows:

\[ \Delta P_{ij}^* = \phi_i + \psi_{ij}, \quad \phi_i \sim N(0, \sigma^2_\phi), \quad \psi_{ij} \sim N(0, \sigma^2_\psi), \quad (5) \]

where \( \phi_i \) is common to all traders and \( \psi_{ij} \) represents the component specific to the \( j \)th trader.

The price change and trading volume can be rewritten by using the variance components model,

\[ \Delta P_i = \phi_i + \bar{\psi}_i, \quad \bar{\psi}_i = \frac{1}{J} \sum_{j=1}^{J} \psi_{ij}, \quad (6) \]

\[ V_i = \frac{\alpha}{2} \sum_{j=1}^{J} |\psi_{ij} - \bar{\psi}_i|. \quad (7) \]

Finally, the following daily joint distribution of returns and the associated trading volume is developed:

\[ \Delta P \mid V \sim N \left( \begin{pmatrix} 0 \\ \mu_v \end{pmatrix}, \begin{pmatrix} \sigma^2_r & \sigma^2_r \\ \sigma^2_r & \sigma^2_v \end{pmatrix} \right), \quad (8) \]

where

\[ \sigma^2_r = (\sigma^2_\phi + \frac{\sigma^2_\psi}{J})I, \]

\[ \mu_v = \frac{\alpha}{2} \sigma_\psi \sqrt{\frac{2}{\pi}} \sqrt{\frac{J-1}{J}} JI, \]
\[ \sigma_v^2 = \left( \frac{\alpha}{2} \right)^2 \sigma_\psi^2 (1 - \frac{2}{\pi}) JI. \]

Here \( \sigma_\psi^2 \) is the degree to which investors disagree with each other, which is our proxy for differences of opinion. To gain further insights into the model, we consider time-varying parameters in our generalized model.

### 1.2. The Generalized Mixture Model

In this section we will develop our generalized mixture model.

It is natural to allow \( \sigma_\psi \) to vary across time with some persistence in the framework of MDH. We assume \( \ln(\sigma_\psi^2) = \eta_t \) follows the linear specification

\[ \ln(\sigma_\psi^2) = \delta_\eta \ln(\sigma_\psi^{2-1}) + \epsilon_{\eta t}, \quad \epsilon_{\eta t} \sim N(0, \sigma_{\eta}^2), \quad (9) \]

\( \Delta P_t \) in equation (8) can be regarded as daily excess return \( r_t - \bar{r} \), where \( \bar{r} \) is the mean of the return. After substituting equation (9) into the joint distribution (8), the resulting specification takes the form

\[ r_t \mid \eta_t, \eta_{t-1} \sim N \left( \begin{pmatrix} 0 \\ \mu_{vt} \end{pmatrix}, \begin{pmatrix} \sigma_{rt}^2 & 0 \\ 0 & \sigma_{vt}^2 \end{pmatrix} \right), \quad (10) \]

where

\[ \sigma_{rt}^2 = I \sigma_\phi^2 + \frac{I}{J} e^\eta, \]

\[ \mu_{vt} = \frac{\alpha}{2} \sqrt{\frac{2}{\pi}} \sqrt{\frac{J - 1}{J}} JI e^{\eta / 2}, \]

and

\[ \sigma_{vt}^2 = \left( \frac{\alpha}{\eta} \right)^2 (1 - \frac{2}{\pi}) JI e^{\eta}. \]
After we further simplify (10) by using the expressions $\beta_1 = I\sigma^2_\phi$, $\beta_2 = \frac{1}{J}$, $\beta_3 = \frac{\alpha}{2} \sqrt{\frac{2}{\pi}} \sqrt{\frac{J-1}{J}} JJ$ and $\beta_4 = (\frac{\alpha}{2})^2 (1 - \frac{2}{\pi}) JJ$, we can obtain the following simplified form

$$r_t \quad | \quad \eta_t, \eta_{t-1} \sim N \begin{pmatrix} 0 \\ \mu_{vt} \end{pmatrix}, \begin{pmatrix} \sigma^2_{rt} & 0 \\ 0 & \sigma^2_{vt} \end{pmatrix} \right)$$

(11)

where

$$\sigma^2_{rt} = \beta_1 + \beta_2 e^{\eta_t},$$

$$\mu_{vt} = \beta_3 e^{\eta_t/2},$$

and

$$\sigma^2_{vt} = \beta_4 e^{\eta_t}.$$

This model allows us to use observable data on return and volume to make inferences about the unobservable variable opinion divergence. We estimate the generalized MDH model using simulated maximum likelihood (SML) and obtain the estimates of daily differences of opinion $\sigma^2_\psi t$ for all the stocks listed in NYSE, AMEX and Nasdaq excluding stocks which do not satisfy certain criteria. Simulated maximum likelihood (SML) was developed by Danielsson and Richard (1993) and applied to the bivariate mixture model by Liesenfeld (1998, 2001). The standard maximum likelihood estimation is infeasible for the mixture model because the latent variables are autocorrelated. We can also estimate the models using GMM proposed by Richardson and Smith (1994). However, as noted by Liesenfeld (1998), the GMM estimators are inefficient because the GMM estimator is based only on the model assumptions about the moment restrictions. The most important reason that we use SML is that it allows the estimation

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6Criteria to exclude some stocks are discussed later in the data section.
of the latent variable, which is \( \eta_t \), and further the function of \( \eta_t \), which is \( \sigma^2_{\psi t} \) in this paper. In this way, we can isolate divergence of opinion from the joint distribution of volume and volatility. The details of the SML estimation procedure are provided in the appendix A. As we are interested in the relationship between monthly differences of opinion and expected returns, we construct the monthly measure of differences of opinion by calculating the monthly average of daily \( \sigma^2_{\psi t} \) as follows:

\[
DIV = \sum_{i=1}^{m} \sigma^2_{\psi t} .
\] (12)

We sort stocks into deciles based on market capitalization, book-to-market and past returns and examine the relationship between divergence of opinion and future asset returns.

II. Properties of Divergence of Opinion

2.1. Data

Our return and volume data are taken from the Center for Research in Securities Prices (CRSP) Daily Stocks Combined File, including all NYSE, AMEX and Nasdaq stocks from August 1, 1962 to December 31, 2003. Real estate investment trusts, stocks of companies incorporated outside United States and closed-end funds are excluded from our analysis. Following Jegadeesh and Titman (2001), we also exclude stocks with share prices less than $5 and greater than $1000 at the beginning of each month, so that our results are not driven by extremely small, illiquid stocks, most of which do not have volume data. Lo and Wang (2002) emphasize the importance of using turnover instead of volume in cross-sectional studies, thus we use daily turnover and

\(^7\)The sample size of the portfolio analysis in Section III only starts from January 1970 because there are not enough stocks in each portfolios for the period 1962-1970.
return to estimate the model. As it is widely documented that turnover and volume have strong trends, we adjust the individual turnover series by stochastic trend components obtained by a two-sided moving average. We construct monthly divergence of opinion series from August 1962 to December 2003 for all stocks in our sample. The data on analysts’ earnings estimates are taken from the Institutional Brokers Estimate System (I/B/E/S). Following DMS, we define dispersion in analysts’ earnings forecasts as the ratio of standard deviation of the estimates to the mean estimate and construct monthly dispersion in analysts’ earnings forecasts from January 1983 to December 2003. Stocks followed by less than two analysts are excluded. The measure of probability of information-based trading (PIN) requires the classification of the number of sells and buys each day. We first retrieve transaction data from the Institute for the Study of Security Markets (ISSM) and Trade And Quote (TAQ) data sets. Then we follow the standard Lee-Ready algorithm (see Lee and Ready (1991)) to classify trades as buys or sells. The estimation of the measure PIN depends on the maximum likelihood estimation. We can only obtain yearly PIN as we need to have at least 60 days with quotes or trades for each estimation of the MLE. Stocks in any year in which there are not at least 60 observations are also excluded from our sample.

2.2. Relation between Divergence of Opinion and Other Variables

In order to show the relationship between our proxy for divergence of opinion and other variables, we run Fama-MacBeth (1973) cross-sectional regressions of DIV against other proxies for divergence of opinion such as dispersion in analysts’ earnings forecasts and turnover, proxy for risk, and PIN (Probability of information-based trading) over the sample size of January 1983 to December 2003. Table I reports the results of

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8See Anderson (1996) for details about it.
9As PIN and dispersion in analysts’ earnings forecasts starts from January 1983 to December 2003, we do the cross-sectional analysis in this sample period.
the regression. The dependent variable is our proxy for divergence of opinion (DIV). The independent variables are market beta (estimated using past 36 to 60 months of data), lagged value of market capitalization, lagged value of book-to-market ratio, turnover, dispersion in analysts’ earnings forecast and PIN.\textsuperscript{10} The results show that divergence of opinion is strongly positively related to market beta. DIV is also significantly positively related with turnover and dispersion in analysts’ earnings forecast. In particular, turnover is the most significant variable among all with t-statistics of 27.2. This further justifies that this new proxy for divergence of opinion actually captures the differences of opinion among all investors. Moreover, DIV does not have a significant relationship with PIN, the private information measure. Therefore, PIN is not the main driving force of the proxy for divergence of opinion. When we run the regression of DIV on market beta, ln(ME), ln(BE/ME) and dispersion, we find that the explanatory power of dispersion for DIV is lower, but still significant at the 10 percent level. We repeat the same regression by adding turnover as an additional independent variable, and find that turnover is still the most significant explanatory variable and the coefficient of dispersion is no longer significant. This indicates that turnover drives out the effect of dispersion on DIV.

2.3. Cross-Sectional Distribution of Divergence of Opinion over Time

Before we investigate the cross-sectional relationship between divergence of opinion and stock returns, it is also interesting to examine the time series patterns of cross-sectional distribution of divergence of opinion.

Figure 1 reports plot of monthly cross-sectional average divergence of opinion over time with the NBER business cycle and 1987 stock market crash marked. The cross-sectional average DIV, which is also defined as aggregate divergence of opinion, shows large time variation. Occasional upward spikes in the figure indicate months in which a

\textsuperscript{10}We construct the market betas exactly in the same way as Fama and French (1992).
stock market crash or economic recession occurred. The largest upward spike in aggregate divergence of opinion occurs in the month of stock market crash, October 1987. The market-wide aggregate liquidity measure in Pastor and Stambaugh (2003) is lowest in the same period, indicating that when divergence of opinion and stock volatility is high, as in 1987 crash, compensations requested by the liquidity provider are greater. Moreover, nearly all the other upward spikes happened in business recessions and stock market downturns. For example, the third largest upward spike is in November 1973, the first full month of the Middle East oil embargo and the subsequent 1973-1974 stock market crash. However, there are some upward spikes that happened during economic booms, such as in February 1976. Therefore, aggregate divergence of opinion does not always coincide with low liquidity and high volatility. The correlation between the aggregate divergence of opinion and the aggregate liquidity measure in Pastor and Stambaugh (2003) is -0.22, confirming the insight obtained from Figure 1. It will be interesting to further analyze the relationship between aggregate divergence of opinion and stock returns. As the main focus of this paper is to examine the effects of divergence of opinion on cross-sectional returns and the crucial role of short-sale constraints in determining the effects, we will leave that analysis to future work.

In Figure 2, we plot the 25th, 50th and 75th percentiles for each month in the sample period for the cross-sectional distribution of DIV. All three percentiles appear to have similar time-varying patterns, with upward spikes in stock market crash and business recessions. The median, the 50th percentile, of the divergence of opinion is around 0.9. The reasonable time variations for all the percentiles and their nearly parallel movements over time show that the upward and downward spikes that we see in Figure 1 are not results of unstable behavior of certain stocks, but is related to the stock market characteristics.
III. Empirical Results

The main objective of this paper is to examine the role of divergence of opinion in affecting future stock returns. In this section, we develop and test our hypothesis that associates the effect of divergence of opinion with different characteristics of stocks for which short-sale constraints are more or less likely to bind.

There is no theoretical consensus on how divergence of opinion affects asset pricing. Miller (1977) combines the effects of short-sale constraints and divergence of opinion on stock prices. He argues that with short-sales constraints, a stock’s price will only reflect the optimistic investors’ valuations, and the valuations of pessimists cannot be reflected in the stock price. This is because the pessimists do not short sell due to the prohibitive costs or difficulty involved in short selling, and the most optimistic investors will bid the price higher than the true value of the stock. Therefore the basic prediction of Miller (1977) is that the greater the disagreements in the valuation of the stock among investors, the lower the subsequent return.

There are two simple assumptions in Miller (1977). First, Miller assumes that investors have different estimates of the stock’s value. Many models note that investors can come to different conclusions about a stock’s fundamental value, even when they read the same public news about the stocks, and show that volume can largely be explained by differences of opinion, e.g. Harris and Raviv (1993); Kandel and Pearson (1995). Second, it requires the existence of short-sales constraints to make sure that pessimistic views will not be reflected in the stock price. This assumption is empirically reasonable for those stocks for which short-sale constraints are more likely to bind.

On the other hand, Hong and Stein (2003) rely heavily on the existence of perfectly rational arbitrageurs who do not face short-sale constraints. Unbiased prices are achieved by assuming that perfectly rational arbitrageurs can costlessly short sell at any time, which can clear the market at a price that is equal to the expected stock valuation.
Rational arbitrageurs without short-sale impediments, recognizing that the true value of a stock is lower than the optimistic investors’ value, will short sell the stock and help push its price back to the true value. The assumptions behind the Hong-Stein model, which predicts unbiased prices even with divergence of opinion, are obviously strong for those stocks that are more difficult to short sell. For stocks with high shorting costs, the price will be biased upwardly when there is high divergence of opinion, because rational arbitrageurs who also need to pay the high shorting costs are not able to correct the over-pricing. However, this model is more appealing for those stocks that are easier to short sell or for which it is easier to find trading options.

We conjecture that both sides have some truth in certain scenarios. The effects of divergence of opinion on future stock returns should be more pronounced for those stocks with short-sale constraints more likely to bind, as argued by Miller. However, for stocks that are easy to short sell, stock prices should be unbiased even when opinions diverge, which is Hong and Stein’s (2003) prediction. Moreover, recent literature in short-sale constraints documents that small, low-book-to-market, and high-momentum stocks are more difficult to short sell. Indeed, it may even be impossible to find a willing lender for these stocks. This paper differentiates the divergence of opinion effects on returns for stocks with differing degree of difficulty associated with completing a short sale transaction by using our new proxy for divergence of opinion to test the following hypothesis: *Divergence of opinion will cause an upward bias in the prices of small, low-book-to-market, and high-momentum stocks, leading to positive return differences between low- and high-divergence stocks.*

We then use our measure for divergence of opinion to test our main hypothesis. We first sort stocks into five portfolios based on divergence of opinion of the previous month and hold the portfolios for one month. The average monthly return for each of the five portfolios is presented in the first column in Table II. The results clearly show

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11In Diamond and Verrecchia (1987), prices are unbiased because rational uninformed arbitrageurs can take into account the effects of short-sale constraints and sets bid and ask prices correctly.
that the lower the divergence of opinion, the higher the subsequent return. When we move from the lowest divergence decile to the highest divergence decile, the average monthly return of the decile goes down from 1.57 percent to 0.92 percent. The return difference between low- and high-divergence deciles is 8 percent annually and is statistically significant. This shows that for all stocks taken together, the results support Miller’s prediction.

To test our hypothesis and see if Miller’s prediction holds for stock groups with short-sales constraints more or less likely to bind, we sort stocks based on certain characteristics: size, book-to-market and momentum. By using this standard approach in asset pricing, we can reduce the variability in returns.

3.1. The Effect of Size on Return Differences between Low- and High-Divergence Stocks

We begin our analysis by first looking at the effect of size on return differences between low- and high-divergence stocks. Each month, we assign stocks into five size quintiles based on market capitalization as of the end of the previous month. Then, within each size group, we further assign stocks into five quintiles based on opinion divergence as of the end of the previous month. After assigning stocks into portfolios, stocks are held for one month. We calculate the equal-weighted monthly average returns of all the stocks in each portfolio.

Table II shows that there is a strong negative relationship between opinion divergence and average returns for small-size and mid-size stocks. The average returns of the portfolio decrease as we move from the divergence of opinion group to the high group for those small-size and mid-size stocks. The effect is not as pronounced among large-size stocks. The return differential between low- and high-divergence groups is largest and significantly positive for the smallest three size groups. Specifically, the re-
turn difference is 17 percent, 15 percent and 6 percent for the smallest three size groups respectively. However, the return difference is either not significant or has the wrong sign for large-size stocks. Small-size stocks are more difficult to short and less likely to have traded options. This result supports our hypothesis that, when opinions diverge, the more likely there are short-sale impediments, the lower the future returns, and the bigger the return difference between low- and high-divergence stock groups.

Table II also presents the monthly average measure of divergence of opinion (DIV). The divergence of opinion is a little bigger for larger-size stocks, which is consistent with the positive sign of the coefficient on ln(ME) in the Fama-Macbeth regression in Table 1. It also indicates that divergence of opinion comes mostly from different interpretations of public information, since for large-size stocks, it is public information that is more important. This further confirms the insignificant role played by PIN in explaining DIV we find in Table I. Moreover, when we compare average DIV of the lowest divergence of opinion group and that of the highest divergence of opinion group for all size groups, it is clear that DIV difference between low- and high-divergence groups has similar magnitude of about -1.1 for all size groups. This also confirms that it is not the DIV differences in the magnitude across the five size groups that causes the bigger divergence of opinion effect on returns for small-size and mid-size stocks.

Fama and French (1996) argue that the three-factor model in Fama and French (1993) captures many of return anomalies. In the three-factor model, $R_m$ is the excess return on the market, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate, SMB (Small Minus Big) is the difference between average return on three small portfolios and the average return on three big portfolios, and HML (High Minus Low) is the difference between average return on two value portfolios and the average return on two growth portfolios. Following Carhart (1997), we include the momentum variable UMD into

\footnote{See Fama and French (1996) for more details about how to construct these factors.}
the three-factor model. UMD is the difference between the past winners with high returns from \( t - 12 \) to \( t - 2 \) and the past losers with low returns from \( t - 12 \) to \( t - 2 \). To further examine if the low divergence of opinion portfolios still outperform high divergence of opinion counterparts for small stocks after we adjust Fama-French risk factors and the momentum factor, we estimate the following four-factor model for each of the twenty five portfolios sorted based on size and \( \text{DIV} \):\(^{13}\)

\[
E(R_{it} - R_{Ft}) = a_i R_m + s_i SML_t + b_i HML_t + m_i UMD_t.
\]

(13)

The estimates of the intercepts in equation (13) are reported in table III.\(^{14}\)

From table III, we can see that within the smallest three size groups, the intercepts, which are also the four-factor adjusted returns, are significantly negative for the highest divergence of opinion groups. This indicates that the four-factor model leaves negative large unexplained return for stocks in the highest divergence of opinion groups for small stocks. Moreover, the four-factor adjusted return differences between the low- and high-divergence groups are still significantly positive for the smallest three size groups with magnitude of 17 percent, 14 percent and 7 percent respectively. Therefore, this result strongly shows that the overpricing caused by divergence of opinion among small-size stocks are not driven out by the Fama-French three factors or momentum factor, and our hypothesis is supported even after we adjust the four factors.

3.2. BE/ME Effect on Return Differences between Low- and High-Divergence Stocks

In this subsection, we investigate whether the return differences between low- and high-divergence stock groups are greater for glamour stocks, which are documented to be

\(^{13}\)The twenty five portfolios are sorted exactly the same way as in Table II.

\(^{14}\)The results for Fama-French three-factor model are similar, therefore we only provide estimates of the four-factor model in this paper.
more difficult to short sell. Since glamour stocks tend to have higher levels of market
capitalization, we control the effect of size by presenting three-way cuts on size, book-
to-market and opinion divergence in Table IV.

We first sort stocks into three size quintiles based on the market capitalization as
of the end of the previous month. Then we sort stocks in each size group into three
additional book-to-market groups. Yearly book equity (BE) for all fiscal years ending
in calendar year $t − 1$ is matched with returns starting in July of year $t$. We then divided
this figure by market capitalization (ME) at month $t − 1$ to obtain the monthly BE/ME
ratio. Finally, stocks in each of the nine size and book-to-market groups are assigned
into three groups based on the measure of divergence of opinion at the previous month.
We run equation 13 for each of the twenty-seven portfolios and report estimates of al-
phas in Table IV. Our results illustrate the strong effect of size on risk-adjusted return
differentials, because the two significant positive risk-adjusted return differentials for
small-size stocks have larger magnitude than those for mid-size. Moreover, they are
glamour stocks. Within the small-size group, the risk-adjusted return differences be-
tween low- and high-divergence group are 23 percent and 14 percent annually for low
and mid book-to-market stocks, respectively. Even for the mid size group, the return
differences between low- and high-divergence group are 7 percent and 5 percent annu-
ally for low and mid book-to-market stocks, respectively. For value stocks, the return
differences are insignificant. This corroborates the view that when divergence of opin-
ion is high, short-selling impediments, reflected in higher loan fees in glamour stocks,
prevent the revelation of pessimistic views of the stock value and result in lower future
returns. However, for large-size stocks and value stocks which are relatively easier to
short sell, the effects are not obvious. This is more consistent with Hong and Stein’s
prediction.
3.3. Momentum Effect on Return Differences between Low- and High-Divergence Stocks

As small-size, high-momentum stocks are difficult to short sell, we employ a three-way cut on size, momentum and DIV to investigate whether for high-momentum stocks, divergence of opinion leads to upwardly biased prices. Momentum is computed based on past returns from $t - 12$ to $t - 2$ as in Fama and French (1996). We first sort stocks into three groups based on the level of market capitalization at the end of the previous month. Then within each size group, stocks are further assigned to three momentum groups. Stocks within each size and momentum group are then sorted into three categories based on divergence of opinion at the previous month.

Risk-adjusted returns of the twenty-seven portfolios are reported in Table V. Within the smallest size group, the annual return differentials between low- and high-divergence stocks for the mid- and high-momentum groups are significantly positive, with magnitude of 11 percent and 26 percent, respectively. Among mid-size stocks, corresponding return differences are 5 percent and 12 percent, respectively. Therefore, this effect is still pronounced, although not as strong as among small-size stocks. However, for the low-momentum group, the return differential is insignificantly negative, except for large-size stocks.\footnote{Sub-period analysis for the twenty-seven portfolios sorted based on size, momentum and DIV, which is not provided in the main body of the paper, shows that this significant negative effect is only driven by the subperiod of 1970-1982.} This corroborates that for small-size stocks and high-momentum stocks, it is more difficult and costly to short sell, and therefore, divergence of opinion causes overpricing.
IV. Robustness Check

In this section, we demonstrate that the overpricing caused by divergence of opinion given short-sale constraints is robust to additional trading strategy.

4.1. Subperiod Analysis

We divide the sample into three sub-samples: 1970-1982, 1983-1992, and 1993-2003. We report the mean monthly return differentials between low- and high-divergence groups for all stocks and each of five size-groups for the three subperiods in Table VI. The first column of the table shows that for all stocks, the return difference declines from a significant 1.03 to an insignificant 0.36 as we progress from the 1970-1982 period to more recent periods. The declining trend is also obvious when we do within-size-group analysis. The return differential is significantly positive for the smallest three size quintiles for the period 1970-1982, with respective magnitudes of 1.85, 1.86 and 0.93. For the period 1983-1992, the return differential is also significant for the smallest three size-groups, with magnitude of 1.40, 0.93, and 0.58 respectively, while for the period 1993-2003, the significant positive return differentials reduce to only the smallest two size-groups. Moreover, within each size-group, the magnitudes of the return differential are lower for recent periods. Specifically, they move down from 1.85 to 1.01 percent when we come from the 1970s to recent periods for smallest-size group, 1.86 to 0.70 for the second-smallest group, and 0.93 to -0.05 for the mid-size group. Short-sale costs have come down over time, leading to less binding short-sale constraints. This subperiod analysis not only shows that our results before are robust across various sub-samples, but it also supports the argument of Miller (1977) that short-sale constraints prevent the revelation of pessimistic valuations of stocks and result in lower future returns.

This result has interesting policy implications. Short-selling by bearish investors is
blamed for the collapse of the stock market bubble, and therefore opponents of short-selling support anti-speculative legislations banning short selling. Proponents of short-selling argue that short-selling by pessimistic investors helps both to avoid wasteful bubbles and to achieve market efficiency. The evidence in this paper shows that divergence of opinion leads to more upwardly biased stock prices when short-sale constraints are more binding, such as in old periods 1970-1989, and for small-size, glamour, and high-momentum stocks. Therefore our results are consistent with the view that we need more shorting activities.\textsuperscript{16} Short-selling can be expensive for certain stocks. Thus, to lower both the direct costs of short-selling and the risk entailed with shorting is one way to ensure that more shorting happens in the market. When stock options are available, pessimists can take "short" positions by buying puts and/or writing calls. Hence introduction of options tend to increase shorting activity.\textsuperscript{17} This also explains our finding that, in recent period stock price is less biased upward for high divergence of opinion stocks.

4.2. Different Lag in Portfolio Formation

To see how long the overpricing for all stocks and different size deciles can last, we assign stocks to portfolios after a wait of several months. Figure 3 presents the mean return differentials between low- and high-divergence groups after waiting for zero to eleven months for all the stocks and for stocks in the five size-groups. S1 denotes the smallest-size group, and S5 the largest. As shown in figure 3, return differentials for all stocks and all size groups decline as the lag becomes longer. Overpricing disappears because as more months have passed before we form the portfolio, investors are better


\textsuperscript{17}Danielson and Sorescu (2001) employ option introduction as proxy for a decline in short-sale constraints.
informed about the stock’s true value, while optimistic investors are becoming disappointed and selling the stocks. The return differential becomes insignificant if the lag is longer than eight months for all stocks, ten for S1, nine for S2, six for S3, and three for S4. The smaller the stock’s market capitalization, the longer it takes for overpricing to disappear. This result, again, is consistent with our hypothesis. These results also shed light on the strategy of investors. As the overpricing caused by divergence of opinion takes several months to disappear, and as it is most difficult or costly to take a short position in those small stocks for which overpricing is most pronounced, it is better to avoid buying overvalued stocks bid up by overconfident investors.

4.3. Cross-Sectional Regressions

As shown before, turnover and dispersion in analysts’ earnings forecasts are significant explanatory variables for our proxy for divergence of opinion. In our Fama-MacBeth cross-sectional regression of returns on other variables, we include these two proxies as independent variables, in addition to size, book-to-market ratio, and various momentum variables that can control for documented anomalies in the literature. The results are reported in Table VI. We can see that the coefficient of our proxy for divergence of opinion is significantly negative for all the specifications. Before we add turnover and dispersion in analysts’ earnings forecasts, our measure of divergence of opinion has negative effects on returns with a significant level of -4.49. After turnover and dispersion in analysts’ earnings forecast are included in the regression, our proxy for divergence of opinion is still significant, while the other two proxies are not. Therefore the effects of our measure on expected return are not driven out by other proxies for divergence of opinion.
4.4. Discussion

Our results in this paper strongly support our hypotheses in section III. While the evidence is consistent with Miller’s prediction, Hong and Stein’s (2003) conclusion is violated for stocks which are difficult to short sell. These stocks include small-cap, low-book-to-market and high-momentum stocks.

Although DMS utilize Miller’s rationale, their findings that the effects of dispersion of opinion among analysts on returns are most pronounced for low-momentum returns and no apparent patterns can be seen for low-book-to-market stocks are in contrast with our evidence. In the context of short-sale constraints, it is hard to explain their results because short-sale constraints are more severe for low-book-to-market and high-momentum stocks. Moreover, they argue that it is any friction (not necessarily short-sale constraints) preventing the revelation of negative valuation that leads to stock overpricing. The friction is referred to the incentive structure of analysts, i.e. when divergence of opinion is high, pessimistic analysts simply stop covering the stock or do not report their forecasts, which leads to upwardly biased prices. Thus, the evidence connecting dispersion effect on returns to short-sale costs is mixed in that article. By contrast, this article provides a sharper test of indication of Miller’s (1977) and Hong and Stein’s (2003) predictions by employing a direct proxy for divergence of opinion among all individual investors and emphasizes the role of short-sale constraints in affecting the relation between opinion divergence and asset returns.

V. Conclusion and Future Work

In this paper we provide a new proxy for divergence of opinion among all individual investors, estimated from the generalized model of Pitts and Tauchen (1983). Our measure of divergence of opinion has significant relations with other proxies for divergence of opinion, such as dispersion in analysts’ earnings forecast and turnover. As a
proxy for divergence of opinion, our measure has several advantages compared to those proxies currently used in the literature. It is isolated from the joint distribution of the volatility and volume and can directly capture the information of divergence of opinion among all individual investors, as opposed to dispersion in analysts’ earnings forecast, which is only a proxy for differences of opinion among analysts, and turnover, which is also a proxy for other variables.

We use our measure DIV to examine the cross-sectional effects of divergence of opinion on future stock returns. Our results support Miller’s prediction that divergence of opinion results in upwardly biased prices when there exist short-sale impediments. Moreover, we devise a sharper test of the effects of divergence of opinion on returns by testing our hypothesis that for small-cap, glamour and high-momentum stocks, the higher the divergence of opinion, the lower the future stock returns. We sort stocks into different classes such as size, book-to-market, and momentum, because those characteristics affect short-sales constraints. We find that stock prices are most upwardly biased, and return differences between low-divergence and high-divergence stock groups are biggest, for small, low-book-to-market and high-momentum stocks. The evidence for small, glamour, and high-momentum stocks is consistent with Miller’s theory, while the Hong and Stein (2003) model’s prediction of unbiased prices is valid for large-cap, value, and low-momentum stocks. Our results also disprove the notion that divergence of opinion can be regarded as a proxy for risk. The results have implications for both policy and investment strategy. For policy makers: the results suggest that we should have less short-sale constraints, not more, to achieve market efficiency. For investors: they should avoid investing in overvalued stocks bid up by optimistic investors.

This paper also suggests several directions for future research. First, we will study how short-sale constraints affect the relationship between returns and divergence of opinion by using a direct measure of short-sale constraints from the equity lending market. In this way, we can estimate quantitatively how much the short-sale constraints
can explain the superior performance of low-divergence-of-opinion stocks. Second, the
time series characteristics of the monthly average of divergence of opinion across all
the stocks, defined as the market-wide aggregate divergence of opinion, show that high
divergence of opinion coincide with the 1987 stock crash and with most business cycle
downturns. It is interesting to explore the relationship between market-wide aggregate
divergence of opinion, aggregate market-wide liquidity, and stock returns in future
work. Finally, another question for future work is to test Hong and Stein’s (2003)
hypothesis that high divergence of opinion should forecast more negative skewness.
While leverage-effects, volatility-feedback, and bubble theories all fail to explain the
underlying mechanism of negatively-skewed market returns, the new proxy for diver-
gence of opinion can potentially lead to an answer to this puzzle.

Appendix A.

A.1 Estimation Method: Simulated MLE

Let \( y_t = \{r_t, v_t\}_{t=1}^T \) denote the matrix of observable variables \( r_t \) and \( v_t \), \( x_t = \{\eta_t\}_{t=1}^T \) denote the vector of latent variables \( \eta_t \) and \( f(y_t, x_t|\theta) \) represent the joint density of \( x_t \) and \( y_t \), where \( \theta \) denotes the vector of parameters to be estimated. The likelihood function of generalized model in section 1.2 associated with the observable variables is given by the following integral

\[
L(\theta; y_t) = \int_{\omega} f(y_t, x_t|\theta) dx_t, \tag{14}
\]

where \( \omega \) is the support over \( R^{2T} \) for models with the dynamic latent variable. In the SML approach, the likelihood (14) is evaluated by a MC technique based on an importance sampling procedure and then maximized to obtain estimates of \( \theta \).
We can factorize \( f(y_t, x_t | \theta) \) into 
\[
k(y_t | x_t, \theta) = \prod_{t=1}^{T} k(y_t | x_t, \theta) \quad \text{and} \quad g(x_t | \theta) = \prod_{t=1}^{T} g(x_t | x_{t-1}, \theta),
\]
where \( k(y_t | x_t, \theta) \) denotes the joint density of observable variables return \( r_t \) and trading volume \( v_t \) conditional on latent variables \( x_t = \{ \eta_t \} \), and \( g(x_t | x_{t-1}, \theta) \) denotes the conditional density of the latent variables given their past values. Then equation (14) can be written as

\[
L(\theta; y_t) = \int_{\omega} k(y_t | x_t, \theta) g(x_t | \theta) dx_t = \int_{\omega} \prod_{t=1}^{T} k(y_t | x_t, \theta) g(x_t | x_{t-1}, \theta) dx_t.
\]

(15)

We know that \( k(y_t | x_t, \theta) \) is bivariate Gaussian and \( g(x_t | x_{t-1}, \theta) \) is univariate Gaussian, so if we regard (15) as expected value of \( k(y_t | x_t, \theta) \) on the distribution \( g(x_t | \theta) \) and simulate \( N \) i.i.d. samplers \( \{ \tilde{x}_{t,n} \}_{n=1}^{N} \) drawn from the distribution \( g(x_t | \theta) \), which we refer to as natural samplers, we can easily obtain a natural MC estimator of \( L(\theta; y_t) \) as following:

\[
\frac{1}{N} \sum_{n=1}^{N} k(y_t | \tilde{x}_{t,n}, \theta).
\]

(16)

However, this MC estimate is based on a sequence of sampling densities \( g \) which do not take into account the fact that the observations of \( r_t \) and \( v_t \) contains critical information on the underlying latent process. It is shown by Danielsson and Richard (1993) that a prohibitively huge simulation sample size \( N \) would be required to obtain an accurate natural MC estimator and the natural MC estimator is highly inefficient. In order to address the inefficiency problem, we follow Liesenfeld and Richard (2001) to apply Efficient Importance Sampling (EIS) that significantly generalized Accelerated Gaussian Importance Sampling (AGIS) procedure developed by Danielsson and Richard (1993). EIS searches for a sequence of samplers that exploits the information on the \( \eta_t's \) conveyed by \( r_t \) and \( v_t \). This procedure is to find an auxiliary Gaussian sampler \( s(x_t | \gamma) \)
indexed by the auxiliary parameter vector $\gamma$ that can minimize the MC sampling variance of the corresponding MC estimator of $L(\theta; y_t)$. For any given value of $\gamma$, using $s(x_t|\gamma)$ equation (15) can be rewritten as

$$L(\theta; y_t) = \int \frac{k(y_t|x_t, \theta)g(x_t|\theta)}{s(x_t|\gamma)} s(x_t|\gamma) dx_t, \quad (17)$$

If we can draw $N$ i.i.d. trajectories from the sampler $s(x_t|\gamma)$, which is referred as importance sampler, then the corresponding MC estimator is

$$\hat{L}_N(\theta; y_t) = \frac{1}{N} \sum_{n=1}^{N} \frac{k(y_t|x_{t,n}, \theta)g(x_{t,n}|\theta)}{s(x_{t,n}|\gamma)}. \quad (18)$$

It follows that the MC sampling variance of importance MC estimator (18) is

$$Var[\hat{L}_N(\theta; y_t)] = \frac{1}{N} Var_s\left[\frac{k(y_t|x_t, \theta)g(x_t|\theta)}{s(x_t|\gamma)}\right]. \quad (19)$$

Note that if the value of the parameter vector $\gamma$ could be found such that the density $s(x_t|\gamma)$ is proportional to $k(y_t|x_t, \theta)g(x_t|\theta)$, then MC sampling variance of the importance MC estimator would be zero. EIS is used to search the parameter vector $\gamma$ which can make the shape of $s(x_t|\gamma)$ match that of $k(y_t|x_t, \theta)g(x_t|\theta)$ as well as possible so that the MC sampling variance of $\hat{L}_N(\theta; y_t)$ can be minimized. However, this is a high-dimensional integration problem, so it is computationally infeasible to solve the least squares optimization problem. It is necessary to break down the high-dimensional optimization problem into a sequence of separate low-dimensional optimization problems for each period $t$ as suggested by the factorization in (15) and the factorization of $s(x_t|\gamma)$ into $\prod_{t=1}^{T} s(x_t|x_{t-1}, \gamma)$. Because the integral of $k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta)$ with respect to $x_t$ depends on $x_{t-1}$ and the integral of $s(x_t|x_{t-1}, \gamma)$ is by definition equal to one, it is impossible to obtain a good match between $k.g$ and $s$ period by period independently. However, we could find a positive functional approximation $p(x_t; a_t)$ for the density $k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta)$, which can be analytically integrable with respect to $\eta_t$. 29
And $s(x_t|x_{t-1}, \gamma)$ can be written as

$$s(x_t|x_{t-1}, \gamma) = \frac{p(x_t; \gamma)}{\chi(x_{t-1}, \gamma)}, \quad \text{where } \chi(x_{t-1}, \gamma) = \int p(x_t; \gamma) d\eta_t.$$  \hfill (20)

It follows that we need to find a class of density kernel $p$ for the auxiliary importance samplers $s$ so that $s$ can be a good functional approximation for $k.g$. A natural choice for $s$ is to use parametric extension the natural samplers $g$, therefore the following parametrization for the density kernel $p$ is suggested

$$p(x_t; \gamma_t) = g(\eta_t|\eta_{t-1}, \theta)\xi(\eta_t, \gamma_t),$$ \hfill (21)

where the auxiliary function $\xi(\eta_t, \gamma_t)$ is a Gaussian density kernel and $\xi(\eta_t, \gamma_t) = \exp(\gamma_0 \eta_t + \gamma_1 \eta_t + \gamma_2 \eta_t^2)$. Note that $p$, which is the multiplication of two density kernels of Gaussian distribution, is closed.

Equation (21) also implies that $p(x_t; \gamma_t)$ is a Gaussian density kernel for $\eta_t$ given $\eta_{t-1}$. If there is a good match between $k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta)$ and $p(x_t; \gamma_t)$, then $\chi(x_{t-1}, \gamma_t)$ will be unaccounted for. Nevertheless, as $\chi(x_{t-1}, \gamma_t)$ does not depend on $\eta_t$, it can be transferred back to the minimization problem for period $t - 1$. Therefore, the following back-recursive sequence of least-squares problems need to be solved:

$$\hat{\gamma}_t = \arg\min_{\gamma_t} \sum_{i=1}^{N} \left\{ \ln[k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta)\chi(x_{t-1}, \gamma_t)] - \ln p(x_t; \gamma) \right\}^2$$ \hfill (22)

Note that as the density kernel $p$ is chosen within the exponential family of distributions as in (21), the least-squares problem in equation (22) is linear in $\gamma_t$. After we obtain $\hat{\gamma}_t$, we can estimate the likelihood function (17) for any given value $\theta$. 

30
A.2 The implementation of EIS for the generalized model

The generalized model characterized by equation (11) assumes return $r_t$ and volume $v_t$ given $\eta_t$ and $\eta_{t-1}$ are Gaussian distributed. Their densities are given by:

$$k(y_t|x_t, \theta) \propto e^{\left(-\frac{r_t^2}{2\sigma_{r_t}^2}\right)}e^{\left(-\frac{(v_t - \mu_{vt})^2}{2\sigma_{vt}^2}\right)} \tag{23}$$

$$g(x_t|\theta) \propto e^{\left(-\frac{2\delta^2\eta_{t-1}^2}{2\sigma_{\eta}^2}\right)} \tag{24}$$

where

$$\sigma_{r_t}^2 = \beta_1 + \beta_2 e^{\eta_t},$$

$$\mu_{vt} = \beta_3 e^\eta/2,$$

and

$$\sigma_{vt}^2 = \beta_4 e^{\eta}.$$

The multiplicative factors which do not depend upon $\eta_t$ are omitted.

Therefore, according to equation (21), the density kernels of the importance samplers have the form

$$p(x_t; \gamma) \propto e^{\left(-\frac{\gamma_1^2}{2\sigma_{\gamma_1}^2}\right)} \times \left\{ e^{\left(-\frac{1}{2}[(\gamma_1^2 - 2\gamma_2_t)\eta_t^2 - 2(\frac{\delta^2\eta_{t-1}}{\sigma_{\eta}^2} + \gamma_1)\eta_t + (\frac{\delta^2\eta_{t-1}}{\sigma_{\eta}})^2])}\right\} \tag{25}$$

Accordingly, the conditional mean and variance of $\eta_t$ on the density kernel $s(x_t|\gamma)$ is the following

$$\mu_{s,t} = \sigma_{s,t}^2 \left(\frac{\delta^2\eta_{t-1}}{\sigma_{\eta}^2} + \gamma_{1,t}\right), \quad \sigma_{s,t}^2 = \frac{\sigma_{\eta}^2}{1 - 2\sigma_{\eta}^2\gamma_{2,t}}, \tag{26}$$
After integrating \( p(x_t; \gamma) \) with respect to \( \eta_t \), we have the following expression for the integrating constant:

\[
\chi(x_{t-1}, \gamma) \propto \exp \left\{ \frac{\mu_{s,t}^2}{2\sigma_{s,t}^2} - \frac{(\delta \eta_{t-1})^2}{2\sigma_{\eta}^2} \right\}
\]  

(27)

To implement EIS, the following procedure is required:

Step 1: Draw \( N \) trajectories of the latent variable \( x_t = \{\eta_t\} \) from the natural sampler \( g \).

Step 2: Solve the back-recursive sequence of least-squares problems defined in (22) to obtain the solutions \( \{\tilde{\gamma}^*_t\} \) using these random draws. For every period \( t = 1, \ldots, T \), based on \( N \) observations run the following linear auxiliary regression:

\[
\ln k_t(y_t|\tilde{\eta}_{t,n}, \theta) + \ln \left( \chi(\tilde{\eta}_{t,n}, \gamma_{t+1}) \right) = \gamma_{0,t} + \gamma_{1,t}\tilde{\eta}_{t,n} + \gamma_{2,t}\tilde{\eta}_{t,n}^2 + \text{residual}
\]  

(28)

Step 3: Based on \( \{\tilde{\gamma}^*_t\} \), determine the sequence samplers \( s(x_t|\gamma^*_t) \) which have conditional mean and conditional variance given in equation (26). Use these new sequence samplers to draw \( N \) new simulated sample \( \{\tilde{\eta}_{t,n}\}_{n=1}^N \).

Step 4: A small number of iterations of step 2 and step 3 are required to obtain the efficient importance samplers.

Step 5: The likelihood function characterized by equation (18) is estimated using the new simulated sample \( \{\tilde{\eta}_{t,n}\}_{n=1}^N \) and estimators for \( \gamma \) are obtained.

More details about implementing EIS to search for \( s(x_t|\gamma) \) and estimating the mixture model are provided by Danielsson and Richard (1993) and Liesenfeld (2001).
References


Figure 1. Cross-Sectional Average of Divergence of Opinion
The figure shows the cross-sectional average of the monthly divergence of opinion across all the stocks in the market over time. The sample size is from August 1962 to December 2003.
Figure 2. Cross-Sectional Distribution of Divergence of Opinion
The figure shows the 25th, 50th and 75th percentiles each month for the cross-sectional distribution of divergence of opinion across all the stocks in the market over time. The sample size is from August 1962 to December 2003.
Figure 3. Plot of Return Differentials for Varying Lags in Portfolio Formation

Stocks are assigned to size deciles based on market capitalization at the end of previous month and then within each size decile, we further assign stocks into divergence of opinion group based on divergence of opinion at the previous month. We wait certain lags before we assign stocks into equal-weighted portfolios and calculate differences in returns between low- and high-divergence of opinion groups. The sample size is February 1970 to December 2003.
Table I  
**FM Regressions of Divergence of Opinion on Lagged Firm Characteristics**  
Fama and MacBeth (1973) cross-sectional regressions are run every month from February 1983 to Dec 2003. The dependent variable is our measure of monthly divergence of opinion (DIV). Beta is estimated using past 60 months of data, lnME is the log of one month lagged ME, lnB/M is the log of one month lag of BE/ME, Disp is dispersion in analysts’ earnings forecasts, Turn is turnover, PIN is probability of information-based trading proposed in Easley, Kiefer, O’Hara and Paperman (1996). The sample size is February 1983 to December 2003.

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<th>Beta</th>
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<th>lnB/M</th>
<th>Disp</th>
<th>Turn</th>
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Table II
Mean Portfolio Returns By Size and Divergence of Opinion
The table presents results for portfolios of stocks sorted by size and divergence of opinions. Size is the market value of equity measured at the end of month $t-1$ and DIV is the proxy for divergence of opinion at month $t-1$. For each month $t$, stocks are sorted into five size groups based on size at the end of previous month. Stocks in each size groups are then sorted into five additional groups based on DIV at the end of the previous month. Stocks are held for one month and portfolio returns are equal-weighted. The table reports the average monthly portfolio returns over period February 1970 to December 2003.

<table>
<thead>
<tr>
<th>DIV Quintiles</th>
<th>All Stocks</th>
<th>S1 (Small)</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5 (Large)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1(low)</td>
<td>1.571</td>
<td>2.180</td>
<td>1.963</td>
<td>1.399</td>
<td>1.238</td>
<td>0.857</td>
</tr>
<tr>
<td>D2</td>
<td>1.360</td>
<td>1.757</td>
<td>1.540</td>
<td>1.377</td>
<td>1.212</td>
<td>1.022</td>
</tr>
<tr>
<td>D3</td>
<td>1.291</td>
<td>1.398</td>
<td>1.332</td>
<td>1.266</td>
<td>1.327</td>
<td>1.235</td>
</tr>
<tr>
<td>D4</td>
<td>1.172</td>
<td>1.077</td>
<td>1.144</td>
<td>1.248</td>
<td>1.242</td>
<td>1.191</td>
</tr>
<tr>
<td>D5(high)</td>
<td>0.918</td>
<td>0.729</td>
<td>0.732</td>
<td>0.863</td>
<td>1.023</td>
<td>1.156</td>
</tr>
<tr>
<td>D1-D5</td>
<td>0.625</td>
<td>1.451</td>
<td>1.231</td>
<td>0.536</td>
<td>0.215</td>
<td>-0.299</td>
</tr>
<tr>
<td>t-stat</td>
<td>(4.309)</td>
<td>(8.823)</td>
<td>(7.352)</td>
<td>(2.923)</td>
<td>(1.352)</td>
<td>(-2.241)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean DIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1(low)</td>
</tr>
<tr>
<td>D2</td>
</tr>
<tr>
<td>D3</td>
</tr>
<tr>
<td>D4</td>
</tr>
<tr>
<td>D5(high)</td>
</tr>
</tbody>
</table>
Table III
Risk-Adjusted Returns of Portfolios Based on Size and DIV

The table presents risk-adjusted returns (relative to a four-factor model
\[ E(R_x - R_f) = a + \beta R_m + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{UMD} UMD_t \]) for portfolios based on size and DIV.

The 5 × 5 equal weighted portfolios sorted based on size and DIV are formed the same way as in Table II. The market premium uses the CRSP NYSE/AMEX/Nasdaq value-weighted index. The variables HML and SMB are created the same methodology as Fama and French (1996). The momentum premium (UMD) is the difference between the return on a portfolio comprised of stocks with high returns and the return on a portfolio comprised of stocks with low returns from t-12 to t-2. The table reports alphas of the regressions over period February 1970 to December 2003.

<table>
<thead>
<tr>
<th>Mean Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size Quintiles</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>DIV Quintiles</td>
</tr>
<tr>
<td>D1(low)</td>
</tr>
<tr>
<td>D2</td>
</tr>
<tr>
<td>D3</td>
</tr>
<tr>
<td>D4</td>
</tr>
<tr>
<td>D5(high)</td>
</tr>
<tr>
<td>D1-D5</td>
</tr>
</tbody>
</table>
Table IV
Mean Portfolio Returns By Size, Book-to-Market, and Differences of Opinion

The table presents risk-adjusted returns (relative to a four-factor model $E(R_u - R_f) = \alpha_u + \beta_u Mkt + \gamma_u SMB_t + \delta_u HML_t + \mu_u UMD_t$) for portfolios based on size, book-to-market and DIV. Size is the market value of equity measured at the end of month t-1, the book-to-market ratio is BE figure for all fiscal years ending in calendar year t-1 matching the returns starting July of year t divided by market capitalization at month t-1 and DIV is proxy for divergence of opinion at month t-1. For each month t, stocks are sorted into three size groups based on size at the end of previous month. Stocks in each size groups are then sorted into three groups based on book-to-market. Each size and book-to-market group is further sorted into three DIV groups. Stocks are held for one month and portfolio returns are equal-weighted. The table reports alphas of the regressions over period February 1970 to December 2003.

<table>
<thead>
<tr>
<th>Size</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE/ME</td>
<td>Low</td>
<td>Mid</td>
<td>High</td>
</tr>
<tr>
<td>DIV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0.91</td>
<td>0.73</td>
<td>0.49</td>
</tr>
<tr>
<td>Mid</td>
<td>-0.03</td>
<td>0.11</td>
<td>0.37</td>
</tr>
<tr>
<td>High</td>
<td>-0.99</td>
<td>-0.41</td>
<td>0.44</td>
</tr>
<tr>
<td>Low-High</td>
<td>1.90</td>
<td>1.14</td>
<td>0.06</td>
</tr>
<tr>
<td>T-stat</td>
<td>(11.4)</td>
<td>(6.86)</td>
<td>(0.33)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE/ME</td>
<td>Low</td>
<td>Mid</td>
<td>High</td>
</tr>
<tr>
<td>DIV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0.65</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>Medium</td>
<td>0.98</td>
<td>0.92</td>
<td>0.95</td>
</tr>
<tr>
<td>High</td>
<td>1.64</td>
<td>1.51</td>
<td>1.57</td>
</tr>
</tbody>
</table>
Table V
Mean Portfolio Returns By Size, Momentum, and Differences of Opinion

The table presents risk-adjusted returns (relative to a four-factor model
\( E(R_p - R_f) = \alpha_p + \beta_M R_M + \beta_S SMB_p + \beta_H HML_p + m_MUMD_p \) for portfolios based on size, momentum and DIV. Size is the market value of equity measured at the end of month t-1, Momentum is based on past returns from t-12 to t-2, and DIV is our proxy for differences of opinion at month t-1. For each month t, stocks are sorted into three size groups based on size at the end of previous month. Stocks in each size groups are then sorted into three groups based on momentum. Each size and momentum group is further sorted into three DIV groups. Stocks are held for one month and portfolio returns are equal-weighted. The table reports alphas of the regressions over period February 1970 to December 2003.

<table>
<thead>
<tr>
<th>Size</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum DIV</td>
<td>Losers</td>
<td>Winners</td>
<td>Losers</td>
</tr>
<tr>
<td>Low</td>
<td>-0.10</td>
<td>0.60</td>
<td>1.57</td>
</tr>
<tr>
<td>Mid</td>
<td>-0.27</td>
<td>0.22</td>
<td>0.63</td>
</tr>
<tr>
<td>High</td>
<td>-0.47</td>
<td>-0.3</td>
<td>-0.63</td>
</tr>
<tr>
<td>Low-High</td>
<td>0.37</td>
<td>0.90</td>
<td>2.20</td>
</tr>
<tr>
<td>T-stat</td>
<td>(2.06)</td>
<td>(5.63)</td>
<td>(12.02)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum DIV</td>
<td>Losers</td>
<td>Winners</td>
<td>Losers</td>
</tr>
<tr>
<td>Low</td>
<td>0.70</td>
<td>0.65</td>
<td>0.68</td>
</tr>
<tr>
<td>Medium</td>
<td>0.96</td>
<td>0.90</td>
<td>0.99</td>
</tr>
<tr>
<td>High</td>
<td>1.58</td>
<td>1.51</td>
<td>1.65</td>
</tr>
</tbody>
</table>

Mean DIV
Table VI
Subperiod Analysis

The table presents average return differential between stocks in the lowest- and highest- dispersion group in each size group over indicated periods. t-statistics are in parentheses. Size is the market value of equity measured at the end of month t-1 and DIV is proxy for divergence of opinion at month t-1. For each month t, stocks are sorted into five size groups based on size at the end of previous month. Stocks in each size groups are then sorted into five additional groups based on DIV at the end of previous month. Stocks are held for one month and portfolio returns are equal-weighted.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>All Stocks</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970-1982</td>
<td>1.03</td>
<td>1.85</td>
<td>1.86</td>
<td>0.93</td>
<td>0.41</td>
<td>-0.46</td>
</tr>
<tr>
<td></td>
<td>(5.91)</td>
<td>(6.63)</td>
<td>(6.74)</td>
<td>(3.31)</td>
<td>(1.72)</td>
<td>(-2.51)</td>
</tr>
<tr>
<td>1983-1992</td>
<td>0.48</td>
<td>1.40</td>
<td>0.934</td>
<td>0.58</td>
<td>0.03</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(3.67)</td>
<td>(5.35)</td>
<td>(4.08)</td>
<td>(2.93)</td>
<td>(0.15)</td>
<td>(1.87)</td>
</tr>
<tr>
<td>1993-2003</td>
<td>0.36</td>
<td>1.01</td>
<td>0.70</td>
<td>-0.05</td>
<td>0.18</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>(1.31)</td>
<td>(3.38)</td>
<td>(2.07)</td>
<td>(-0.13)</td>
<td>(0.50)</td>
<td>(-0.20)</td>
</tr>
<tr>
<td>1970-2003</td>
<td>0.625</td>
<td>1.45</td>
<td>1.23</td>
<td>0.54</td>
<td>0.22</td>
<td>-0.30</td>
</tr>
<tr>
<td></td>
<td>(4.309)</td>
<td>(8.82)</td>
<td>(7.35)</td>
<td>(2.92)</td>
<td>(1.35)</td>
<td>(-2.24)</td>
</tr>
</tbody>
</table>
Table VII

Fama-McBeth Regressions of the Cross Section of Returns on beta, Size, Book-to-Market Equity, Divergence of Opinion, Dispersion in Analysts’ Earnings Forecast and Turnover

Fama and MacBeth (1973) cross-sectional regressions are run every month from February 1983 to December 2003. The dependent variable is expected returns of all the individual stocks in the market. Beta is estimated following Fama and French (1992) using past 60 months of data, lnME is the log of ME at t-1, lnB/M is the log of BE/ME at t-1, $ret_{t-12} - ret_{t-1}$ is momentum at t-1, $ret_{t-1}$ and $ret_{t-36} - ret_{t-13}$ are to capture short-term and long-term effects, DIV is our proxy for divergence of opinion, Disp is dispersion in analysts’ earnings forecasts at t-1 and Turn is turnover at t-1. The table reports coefficients of the independent variables over period February 1970 to December 2003.

<table>
<thead>
<tr>
<th>Beta</th>
<th>ln(ME)</th>
<th>ln(BE/ME)</th>
<th>$ret_{t-12} - ret_{t-1}$</th>
<th>$ret_{t-1}$</th>
<th>$ret_{t-36} - ret_{t-13}$</th>
<th>DIV</th>
<th>Disp</th>
<th>Turn</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.011</td>
<td>0.035</td>
<td>0.180</td>
<td>0.122</td>
<td>-0.028</td>
<td>-0.105</td>
<td>-0.078</td>
<td>-0.078</td>
<td></td>
</tr>
<tr>
<td>(-0.033)</td>
<td>(0.806)</td>
<td>(3.497)</td>
<td>(6.449)</td>
<td>(-6.736)</td>
<td>(-4.295)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.025</td>
<td>0.038</td>
<td>0.178</td>
<td>0.122</td>
<td>-0.029</td>
<td>0.098</td>
<td>-0.078</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.074)</td>
<td>(0.868)</td>
<td>(3.365)</td>
<td>(6.380)</td>
<td>(-6.792)</td>
<td>(3.680)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.047</td>
<td>0.0004</td>
<td>0.050</td>
<td>0.105</td>
<td>-0.036</td>
<td>-0.113</td>
<td>-0.048</td>
<td>-0.114</td>
<td>0.013</td>
</tr>
<tr>
<td>(0.149)</td>
<td>(0.008)</td>
<td>(0.814)</td>
<td>(4.071)</td>
<td>(-7.039)</td>
<td>(-3.714)</td>
<td>(-1.865)</td>
<td>(-1.357)</td>
<td>(1.090)</td>
</tr>
</tbody>
</table>