Divergence of Opinion, Arbitrage Costs and Stock Returns

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Abstract

We develop a new proxy for divergence of opinion and examine how divergence of opinion affects cross-sectional asset returns for stocks with different arbitrage costs. We generalize Tauchen and Pitts’ (1983) Mixture of Distribution Hypothesis (MDH), which links asset volume and volatility in a way that derives a proxy for divergence of opinion among all investors. In our empirical analysis, we establish that the negative relation between divergence of opinion and future returns are especially pronounced for stocks with higher arbitrage costs, such as idiosyncratic risks, short sale costs, and other transaction costs.

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1. Introduction

How does divergence of opinion affect stock returns? In a seminal paper, Miller (1977) argues that divergence of opinion leads to lower returns when there are short sales constraints. In contrast, Hong and Stein (2003) argue that the stock prices aggregate all investors’ valuations in an unbiased way and are thus not affected by divergence of opinion. Their conclusion is based on the strong assumption that timely and frictionless arbitrage is available in the economy.\(^1\)

We conjecture that there is some truth to both sides of this argument when we consider the different level of arbitrage risks for different stocks. On the one hand, stocks with higher arbitrage costs, including short-sale costs, and higher idiosyncratic risk, tend to be more difficult and costly to arbitrage. In those instances, divergence of opinion will have a significant effect. On the other hand, stocks with lower arbitrage costs are much easier to short sell, so, in those instances, divergence of opinion does not have a significant effect. In this paper, we test the above hypothesis and examine the effect of divergence of opinion on cross-sectional stock returns by sorting stocks according to the proxies for short sale cost, other transaction costs and idiosyncratic risk.

We first derive a new proxy for divergence of opinion among all individual investors, based on Tauchen and Pitts’ (1983) Mixture of Distribution Hypothesis (MDH). Prior empirical studies of the divergence effect on equity returns have produced mixed\(^1\)Some theoretical work (e.g., Merton,1987), indicates that divergence of opinion has a positive relationship with future asset returns.
results, partly because the literature fails to identify a reliable proxy for it among all individual investors. Our measure of divergence of opinion (DIV) is strongly and positively related to the existing proxies for divergence of opinion, such as analysts’ earnings forecasts and turnover. Barron, Kim, Lim, and Stevens (1998) argue that the dispersion in analysts’ forecasts is a poor proxy of differences of opinion. Our measure captures the divergence of opinion among all individual investors instead of merely among financial analysts, who may issue biased opinions and herd among themselves due to conflicts of interest and concerns for their job security. Moreover, some investors do not follow analysts’ forecasts, and even those who do may have their own way of aggregating this information. This makes dispersion in analysts’ earnings forecasts a noisy and imprecise proxy for divergence of opinion for all investors. Our proxy is also a cleaner measure of individual investors’ divergence of opinion than asset turnover, because the latter is also used as a proxy both for asset liquidity and for information-induced price adjustment speed. Finally, our measure goes back to January 1970, enabling us to investigate the effect of divergence of opinion on equity returns over a longer sample period.

Using this new measure of divergence of individual investors’ opinions, we test our hypothesis and examine how divergence of opinion affects equity returns when stocks

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2Michaely and Womack (1999), Bradshaw, Richardson, and Sloan (2003), and Jegadeesh, Kim, Krische, and Lee (2004) indicate that the incentive structure of analysts induces them to give positive reports in order to win investment bank deals. Scherbina (2004) documents that the bias in analysts’ earnings forecasts tends to be larger, when analysts exhibit greater divergence of opinion about the earnings.

3Begnoli, Mark, Beneish and Watts (1999) document that investors have been following online unofficial “whisper” forecasts in recent years.

are sorted according to several proxies for their arbitrage costs. Shleifer and Vishny (1997) argue that arbitrage is costly for certain stocks and too high a cost can deter arbitrage. This implies that when the arbitrage costs are too high, it is hard or impossible for pessimistic investors to short sell the stocks. Short sale constraint is only one type of arbitrage costs those short sellers face. Other types of costs include transaction costs and idiosyncratic risk. The proxy we use for short sale constraint is the ratio of short-interest to institutional-ownership, which is grounded in the widely documented evidence that stocks with high short interest and low institutional ownership are more difficult to short sell (e.g., Asquith, Pathak and Ritter, 2005). Other proxies for transaction costs are price, dollar volume, frequency of zero return days (Zerofreq) and Amihud’s (2002) illiquidity measure. The proxy for idiosyncratic risk is the historical standard deviation of the residuals of the four-factor model.

The empirical results support our hypothesis. We find that stocks with high divergence of opinion tend to be overpriced and that the average four-factor risk-adjusted return differences between low-divergence and high-divergence stocks are significantly positive, with the highest decile of DIV outperforming the lowest decile of DIV by 6 percent annually. Consistent with our hypothesis, these effects of divergence are concentrated in stocks with higher transaction costs and arbitrage risks. When we sort all stocks into five equal groups based on the proxy for stocks’ short sale costs, we find that the low divergence of opinion stocks outperform the high divergence of opinion stocks by over 11 percent per year in the highest group after the risk adjustment. In addition, we find that small-size, low-price, illiquid stocks and stocks with higher
frequency of zero return days are most strongly affected by divergence of opinion in
terms of both economic and statistical significance. We also find that the overpricing
caused by divergence of opinion is greater for stocks with higher idiosyncratic risks.
Specifically, stocks with low divergence of opinion outperform those with high diver-
gence of opinion by 26 percent and 10 percent per year for the two groups with the
highest idiosyncratic risk, and our results show the opposite effects for the groups with
the lowest idiosyncratic risk. Moreover, we carry out a Fama-MacBeth (1973) cross-
sectional regression, where the cross-sectional expected returns of all individual stocks
are regressed on DIV, the arbitrage cost measures, and the interaction of DIV and those
arbitrage cost measures, controlling for market beta, size, book-to-market, and past re-
turns. Compared to other types of arbitrage costs, idiosyncratic risk plays a stronger
role on the divergence of opinion effect. This suggests that idiosyncratic risk is an im-
portant deterrent to the arbitrage activity of short sellers and can explain the existence
of the stock overpricing caused by divergence of opinion.

In the robustness analysis, we carry out subperiod analysis for three sample periods:
significant effects on returns during all three subperiods for stocks with high idiosyn-
cratic risk, high illiquidity and high price. The decline in magnitude in recent periods
suggests that the effect of divergence of opinion on equity returns has decreased as the
costs or difficulties in short selling stocks have gradually declined. However, even for
most the recent periods, the negative effects of opinion divergence on returns are still
concentrated in stocks with lower arbitrage costs.
Our research contributes to the literature in several ways. First, we provide a more direct and cleaner measure of the divergence of opinion among all individual investors. Second, the existing empirical studies in this area generate mixed results. For example, Gragg and Malkiel (1982) report a positive relationship between future returns and dispersion of forecasts among a subset of analysts. Lee and Swaminathan (2000) find turnover has a negative relationship to future returns. Diether, Malloy, and Scherbina (DMS) (2002) find that high dispersion in analysts’ earnings forecasts is associated with low future returns. Doukas, Kim, and Pantzalis (2006) employ an alternative measure of diversity based on Barron, Kim, Lim, and Stevens’ (1998) approach and find that stock returns are positively associated with divergence of opinion. However, current tests don’t provide clear evidence linking this relation to arbitrage costs though conclusions from Miller (1977) and Hong and Stein (2003) largely depend on the assumption about the level of arbitrage risks of the stocks. Using more direct measures of arbitrage costs, we find that when arbitrage is costly, the negative effect of divergence of opinion on stock returns is high, supporting Miller (1977). The paper most closely related to our work is Boehme, Danielsen, and Sorescu (2006). They attempt to examine the relation between divergence of opinion and stock returns when there are short-sale constraints. However, they mainly focus on short-sale costs. Our results show that besides short-sale cost, other arbitrage costs, such as other transaction cost and idiosyncratic risk, also play an important role in the effect of divergence of opinion on stock returns.

Finally, this paper also contributes to the growing body of literature on costly arbitrage. Pontiff (1996) finds that the cost of arbitrage causes the mispricing of closed-end
funds. Ali, Hwang and Trombley (2003) document a relation between the book-to-market effect and arbitrage costs. Wurgler and Zhuravskaya (2002) and Kumar and Lee (2006) find that the deviation of stock price from fundamentals is closely related to the difficulty in arbitrage. We add to the literature by showing that arbitrage cost is important for the negative relation between divergence of opinion and stock returns.

The paper comprises five sections. In Section 2, we discuss the theories that explore the relationship between divergence of opinion and equity return and develop our hypothesis. Section 3 provides the basic model of Tauchen and Pitts (1983) and further generalizes it into a model with time-varying divergence of opinion. Section 4 describes the data, provides statistics that show the relationship between DIV and the existing proxies, and analyzes the cross-sectional distribution of divergence of opinion over time. Section 5 presents the empirical analysis of the effect of divergence of opinion and equity returns and various robustness tests. Section 6 concludes and proposes directions for future study.

2. Theoretical background and hypothesis development

There is no theoretical consensus on how divergence of opinion affects asset pricing. One line of thought stems from Miller (1977), who combines the effects of short-sale constraints and divergence of opinion on stock prices. He argues that, with short-sales constraints, a stock’s price will reflect only the optimistic investors’ valuations and not pessimistic investors’ valuations, because pessimists face the prohibitive costs or other difficulties in short selling; hence the most optimistic investors will bid the price higher than the true value of the stock. The basic prediction of Miller (1977) is that the
greater the disagreements about the valuation of the stock among investors, the lower the subsequent return.

This argument involves two simple assumptions. The first is that investors have different estimates of the stock’s value. Many models note that investors can draw different conclusions about a stock’s fundamental value, even when they read the same public news about the stocks, and they show that volume can be explained largely by differences of opinion (e.g., Harris and Raviv, 1993; Kandel and Pearson, 1995). The second assumption requires the existence of short-sales constraints to make sure that pessimistic views will not be reflected in the stock price. In addition to short-sale constraints, which in the literature refer to lending fees, the difficulty of borrowing the stock to be short, and the risk of short squeeze, short selling involves other arbitrage costs, including idiosyncratic risks and transaction costs. Therefore, if we consider all the arbitrage costs the short sellers face when they feel pessimistic about certain stocks, the second assumption is empirically reasonable for those stocks with higher arbitrage costs involved in short selling such as idiosyncratic risks, short-sale cost and other transaction costs.

Hong and Stein (2003) provide a theory that relies heavily on the existence of perfectly rational arbitragers who do not face arbitrage costs. Unbiased prices are achieved by assuming that perfectly rational arbitragers can costlessly short sell at any time, which can clear the market at a price that is equal to the expected stock valuation. Rational arbitragers without arbitrage impediments in short selling, recognizing that

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\(^5\) Geczy, Musto and Reed (2002) find that short-sale costs alone can only partly explain the return anomalies involved in short-selling strategies.
the true value of a stock is lower than the optimistic investors’ value, will short sell the stock and help push the price back to its true value. The assumptions behind this model, which predicts unbiased prices even with divergence of opinion, are obviously strong for those stocks with lower arbitrage costs. For stocks with higher arbitrage costs, the price will be biased upward when there is high divergence of opinion, because rational arbitrageurs who also need to pay the high arbitrage costs are not able to correct the over-pricing. Therefore, this model is more appealing for those stocks that are easier and less costly to arbitrage.

We conjecture that both sides have some truth in certain scenarios. The effects of divergence of opinion on future stock returns should be more pronounced for those stocks with higher arbitrage costs including direct short-sale cost, other transaction costs and idiosyncratic risk. However, for stocks that are easier and less costly to short sell, stock prices should be unbiased even when opinions diverge. This paper differentiates the divergence of opinion effects on returns according to the degree of difficulty associated with completing a short sale transaction and it uses our new proxy for divergence of opinion to test the following hypothesis: Divergence of opinion will cause an upward bias in the prices of stocks with higher arbitrage costs, leading to positive return differences between low- and high-divergence stocks.

6In Diamond and Verrecchia (1987), prices are unbiased because rational uninformed arbitrageurs can take into account the effects of short-sale constraints and set bid and ask prices correctly.
3. Methodology

3.1. Basic model of Tauchen and Pitts (1983)

The mixture of distribution hypothesis (MDH), proposed by Clark (1973) and further developed by Tauchen and Pitts (1983), states that there is a common information flow that directs both the volume and volatility of the equity. In Tauchen and Pitts (1983), there are $J$ active traders who take either long or short positions in a single futures contract. Within each day, the market goes through a sequence of Walrasian equilibria, each of which is initiated by new information to the market. There are totally $I$ equilibria within the day $t$. News arrives periodically within the day, causing the traders with diverse beliefs to revise their expectations. Each news arrival leads to a new round of trading until the market clears and a new equilibrium is reached.

Suppose $P_{ij}^*$ is the $j$th trader’s reservation price at the $i$th within-day equilibrium, and $P_i$ is the market price at the $i$th within-day equilibrium. Let $Q_{ij}$ denote the desired position of the $j$th trader at the time of $i$th within-day equilibrium. A positive value for $Q_{ij}$ represents a desired long position in the contract and a negative value represents a desired short position. Then at the time of $i$th within-day equilibrium, the desired position $Q_{ij}$ of the $j$th trader is given by the following linear relation

$$Q_{ij} = \alpha [P_{ij}^* - P_i], \quad (j = 1, 2, \ldots J),$$

(1)

where $\alpha > 0$ is constant. The reservation price for each of $J$ active traders, $P_{ij}^*$, differs from the current market quotation. Because traders have different expectations about the future and different needs to transfer risk through the market, each trader’s
own reservation price, $P_{ij}^*$, is different from each other. For nonactive traders, they use market quotation as their reservation price. For simplicity, the number of total traders $J$ is fixed with the day.

The market clearing condition requires $\frac{1}{J} \sum_{j=1}^{J} Q_{ij} = 0$. This together with equation (1) implies that the market price $P_i$ needs to be expressed as

$$P_i = \frac{1}{J} \sum_{j=1}^{J} P_{ij}^*. \quad (2)$$

Next, consider the movement from the $(i-1)$st to the $i$th within-day equilibrium. The traders change their reservation price when a piece of news arrives to the market. This results in the change in each trader’s reservation price $\Delta P_{ij}^*$ and also the change in the market price $\Delta P_i$ because the market price change is the average of the change in each trader’s reservation price according to equation (2). By definition, the associated volume of trading $V_i$ is the one-half the sum of the absolute values of changes in the traders’ positions. Thus the price change from the $(i-1)$st to the $i$th within-day equilibrium and trading volume at the $i$th equilibrium can be written:

$$\Delta P_i = \frac{1}{J} \sum_{j=1}^{J} \Delta P_{ij}^*, \quad (3)$$

$$V_i = \frac{1}{2} \sum_{j=1}^{J} |Q_{ij} - Q_{i-1,j}| = \frac{\alpha}{2} \sum_{j=1}^{J} |\Delta P_{ij}^* - \Delta P_{i-1}|. \quad (4)$$

They further assume a variance-components model and decompose $\Delta P_{ij}^*$ as follows:

$$\Delta P_{ij}^* = \phi_i + \psi_{ij}, \quad \phi_i \sim N(0, \sigma^2_\phi), \quad \psi_{ij} \sim N(0, \sigma^2_\psi). \quad (5)$$
where $\phi_i$ is common to all traders and $\psi_{ij}$ represents the component specific to the $j$th trader. $\phi_i$ and $\psi_{ij}$ are mutually independent both across traders and through time. When traders react nearly unanimously to the new information, the realization for the common component relative to the specific component is large. On the other hand, when traders react differently to the information, the realization of common factor is small relative to the specific component. The variance of $\psi_{ij}$, $\sigma^2_\psi$, captures mainly the degree to which people disagree with each other cross-sectionally, because the model assume the components are mutually serially independent through time. This provides a measure for divergence of opinion. Now the question is how to estimate the this measure for divergence of opinion.

The price change and trading volume can be rewritten by using the variance components model,

$$\Delta P_i = \phi_i + \bar{\psi}_i,$$

$$\bar{\psi}_i = \frac{1}{J} \sum_{j=1}^{J} \psi_{ij},$$

$$V_i = \frac{\alpha}{2} \sum_{j=1}^{J} |\psi_{ij} - \bar{\psi}_i|.$$

It is interesting to see that the common component $\phi_i$ does not appear in the volume equation, indicating that it plays no role in the generation of trading volume. In contrast, the firm specific factor $\psi_{ij}$ play the major role in the trade generation. This is consistent with Kandel and Pearson (1995), which find that there is a significant abnormal volume even when prices do not change in response to the announcement and argue that it is likely due to the differences in interpretation of public information by investors.
Given that the number of within-day equilibrium $I$ is random for each day because the number of new pieces of information arriving to the market each day varies significantly, by summing the within-day price change and volume, the change of market price and volume can be written as:

$$
\Delta P = \sum_{i=1}^{I} \Delta P_i, \quad \Delta P_i \sim N(0, \sigma_r^2)
$$

$$
V = \sum_{i=1}^{I} V_i, \quad V_i \sim N(\mu_v, \sigma_v^2)
$$

Assuming the daily price change and trading volume are mixtures of independent normals with the same mixing variable $I$, conditional on $I$ the distribution of $\Delta P$ and $V$ are the following:

$$
\Delta P_i \sim N(0, \sigma_r^2 I)
$$

$$
V_i \sim N(\mu_v I, \sigma_v^2 I)
$$

Finally, the following daily joint distribution of returns and the associated trading volume is developed:

$$
\frac{\Delta P}{V} \mid I \sim N \left( \begin{pmatrix} 0 \\ \mu_v I \end{pmatrix}, \begin{pmatrix} \sigma_r^2 I & \sigma_r^2 I \\ \sigma_r^2 I & \sigma_v^2 I \end{pmatrix} \right) ,
$$

where

$$
\sigma_r^2 = (\sigma_o^2 + \frac{\sigma_v^2}{J}),
$$
\[ \mu_{\psi} = \frac{\alpha}{2} \sigma_{\psi} \sqrt{\frac{2}{\pi}} \sqrt{\frac{J - 1}{J}} J , \]
and
\[ \sigma_{\psi}^2 = \left( \frac{\alpha}{2} \right)^2 \sigma_{\psi}^2 (1 - \frac{2}{\pi}) J . \]

In order to get time-series estimates of the proxy for divergence of opinion, I consider time-varying parameters in the following generalized model.

3.2. The generalized mixture model

It is natural to allow \( \sigma_{\psi} \) to vary across time with some persistence in the framework of MDH. We assume \( \ln(\sigma_{\psi t}^2) = \eta_t \) follows the linear specification

\[ \ln(\sigma_{\psi t}^2) = \delta \eta_{t-1} + \epsilon_{\eta t}, \quad \epsilon_{\eta t} \sim N(0, \sigma_{\eta t}^2) , \quad (9) \]

\( \Delta P_t \) in equation (8) can be regarded as the daily excess return, \( r_t - \bar{r} \), where \( \bar{r} \) is the mean of the return. After substituting equation (9) into the joint distribution (8), the resulting specification takes the form

\[ r_t \mid \eta_t, \eta_{t-1} \sim N \left( \begin{pmatrix} 0 \\ \mu_{\psi t} \end{pmatrix} , \begin{pmatrix} \sigma_{r t}^2 & 0 \\ 0 & \sigma_{\psi t}^2 \end{pmatrix} \right) , \quad (10) \]

where

\[ \sigma_{r t}^2 = I \sigma_{\phi}^2 + \frac{I}{J} \epsilon_{\eta} , \]
\[ \mu_{\psi t} = \frac{\alpha}{2} \sqrt{\frac{2}{\pi}} \sqrt{\frac{J - 1}{J}} J I e^{\eta t / 2} , \]
and

\[ \sigma_{\psi t}^2 = \left( \frac{\alpha}{2} \right)^2 \frac{1}{J} (1 - \frac{2}{\pi}) J I e^{\eta t} . \]
After we further simplify (10) by using the expressions $\beta_1 = I\sigma_0^2$, $\beta_2 = \frac{1}{J}$, $\beta_3 = \frac{\alpha_2}{2}\sqrt{\frac{2}{\pi}}\sqrt{\frac{J-1}{J}}JJ$ and $\beta_4 = (\frac{\alpha_2}{2})^2(1 - \frac{2}{\pi})JJ$, we can obtain the following simplified form

$$r_t \mid \eta_t, \eta_{t-1} \sim N \left( \begin{pmatrix} 0 \\ \mu_{vt} \end{pmatrix}, \begin{pmatrix} \sigma_{rt}^2 & 0 \\ 0 & \sigma_{vt}^2 \end{pmatrix} \right)$$

(11)

where

$$\sigma_{rt}^2 = \beta_1 + \beta_2 e^{\eta_t},$$

$$\mu_{vt} = \beta_3 e^{\eta_t/2},$$

and

$$\sigma_{vt}^2 = \beta_4 e^{\eta_t}.$$ 

This model allows us to use data on observable variables, namely, returns and volume, to draw inferences about the unobservable variable, opinion divergence. We estimate the generalized MDH model using simulated maximum likelihood (SML) and obtain the estimates of daily differences of opinion $\sigma_{vt}^2$ for all the stocks listed in the NYSE, AMEX and Nasdaq excluding stocks that do not satisfy certain criteria. $^7$ SML was developed by Danielsson and Richard (1993) and applied to the bivariate mixture model by Liesenfeld (1998, 2001). The standard maximum likelihood estimation is infeasible for the generalized mixture model because the latent variables are autocorrelated. We can also estimate the models using GMM proposed by Richardson and Smith (1994). However, as noted by Liesenfeld (1998), the GMM estimators are inefficient.

$^7$Criteria to exclude some stocks are discussed later in section 3.1.
because they are based only on the model assumptions about the moment restrictions. The most important reason that we use SML is that it allows the estimation of the latent variable, which is $\eta_t$, as well as function of $\eta_t$, which is $\sigma^2_{\psi_t}$ in this paper. In this way, we can isolate divergence of opinion from the joint distribution of volume and volatility. The details of the SML estimation procedure are provided in appendix A. As we are interested in the relationship between monthly differences of opinion and expected returns, we construct the monthly measure of differences of opinion by calculating the monthly average of daily $\sigma^2_{\psi_t}$ as follows:

$$DIV = \sum_{i=1}^{m} \sigma^2_{\psi_t}.$$  \hspace{1cm} (12)

We sort stocks into deciles based on market capitalization, book-to-market, and past returns and examine the relationship between divergence of opinion and future asset returns.

4. Properties of divergence of opinion

4.1. Data

Our return and volume data are taken from the Center for Research in Securities Prices (CRSP) Daily Stocks Combined File, including all NYSE, AMEX, and Nasdaq stocks from August 1, 1962 to December 31, 2003.\footnote{The sample size of the portfolio analysis in Section 5 starts only from January 1970 because there are not enough stocks in each portfolio for the period 1962-1970.} Real estate investment trusts, stocks of companies incorporated outside United States and closed-end funds are excluded from our analysis. Following Jegadeesh and Titman (2001), we also exclude stocks with...
share prices less than $5 and greater than $1000 at the beginning of each month, so that our results are not driven by extremely small, illiquid stocks, most of which do not have volume data. Lo and Wang (2002) emphasize the importance of using turnover instead of volume in cross-sectional studies; therefore we use daily turnover and return to estimate the model. As it is widely documented that turnover and volume have strong trends, we adjust the individual turnover series by stochastic trend components obtained by a two-sided moving average.\(^9\) We construct monthly divergence of opinion series from August 1962 to December 2003 for all stocks in our sample. The data on analysts’ earnings estimates are taken from the Institutional Brokers Estimate System (I/B/E/S). Following DMS, we define dispersion in analysts’ earnings forecasts as the ratio of the standard deviation of the estimates to the mean estimate and construct monthly dispersion in analysts’ earnings forecasts from January 1983 to December 2003. Stocks followed by fewer than two analysts are excluded. The measure of probability of information-based trading (PIN) requires classifying of the number of sells and buys each day. We first retrieve transaction data from the Institute for the Study of Security Markets (ISSM) and Trade And Quote (TAQ) data sets. Then we follow the standard algorithm (from Lee and Ready (1991)) to classify trades as buys or sells. The estimation of the measure PIN depends on the maximum likelihood estimation. We can obtain only yearly PIN, as we need to have at least 60 days with quotes or trades for each estimation of the MLE. Stocks in any year in which there are not at least 60 observations are also excluded from our sample.

\(^9\)See Andersen (1996) for details.
4.2. Proxies for arbitrage costs

Our proxy for short sale cost is short-interest-to-institutional-ownership ratio (SI/IO). We obtain the institutional ownership data from Thomas Financial from 1980 to 2003 and the short interest data from the NYSE from 1991 to 2003, so our measure SI/IO runs from 1991/2003. In addition to short sale cost, arbitrage costs also include other transaction costs such as the direct transaction cost and indirect cost which measures how quickly the transaction can be completed and how large the adverse price effects are. Our proxies for other transaction costs include price, dollar volume, frequency of zero return days, and Amihud’s (2002) illiquidity measure. Several studies including Bhardwaj and Brooks (1992), Blume and Glodstein (1992), Pontiff (1996) and Ali, Hwang, and Trombley (2003), suggest price is inversely related to direct transaction cost such as bid-ask spreads and brokerage commissions, and, therefore, price can be a proxy for transaction cost. We obtain our measure for price from the monthly closing price per share from CRSP over the period 1970-2003. Regarding volume, other studies argue that it indicates the indirect cost and can proxy for liquidity (e.g., Kyle, 1985; Foster and Viswanathan, 1990; Spiegel and Wang, 2006). Therefore, we use volume, which is obtained by averaging daily volumes for each month, to proxy the indirect transaction cost. It is widely documented that completing transactions for small, illiquid stocks is a slow process and that the adverse price effects for those stocks are bigger. Therefore we also use Amihud (2002)’s illiquidity measure as a proxy for transaction costs, which we obtain by dividing the absolute value of daily returns by daily dollar volume and then averaging them over each month for the period 1970 to 2003. Fol-
ollowing Ali et. al. (2003), our comprehensive measure for transaction costs is zerofreq, which is the number of zero return days in the past 12 months. Stocks with high transaction costs tend to have more zero return days because investors will trade only when the profit outweighs the transaction cost (e.g., Lesmond, Ogden and Trzcinka, 1999). Our proxy for idiosyncratic risk is obtained by regressing an individual stock’s return on the four factors over the previous 60 months and calculating the standard deviation of the residuals. The stocks must have at least 12 months of past return data to be included in our sample.

4.3. Relation between divergence of opinion and other variables

In order to show the relationship between our proxy for divergence of opinion and other variables, we run Fama-MacBeth (1973) cross-sectional regressions of DIV against other proxies for divergence of opinion, such as dispersion in analysts’ earnings forecasts and turnover, proxies for risk, and PIN over the sample period of January 1983 to December 2003. 10 Table 1 reports the regression results. The dependent variable is our proxy for divergence of opinion (DIV). The independent variables are market beta (estimated using past 36 to 60 months of data), lagged value of market capitalization, lagged value of book-to market ratio, turnover, dispersion in analysts’ earnings forecasts and PIN. 11 The results show that divergence of opinion is strongly positively related to market beta. DIV is also significantly positively related with turnover and dispersion in analysts’ earnings forecasts. In particular, turnover is the most significant

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10 This is the period when PIN and dispersion in analysts’ earnings forecasts began.
11 We construct the market betas in exactly the same way as Fama and French (1992).
variable of all, with a t-statistic of 27.2. This further strengthens the case that this new proxy for divergence of opinion actually captures the differences of opinion among all investors. Moreover, DIV does not have a significant relationship with PIN, the private information measure; therefore, PIN is not the driving force of the proxy. When we run the regression of DIV on market beta, ln(ME), ln(BE/ME) and dispersion, we find that the explanatory power of dispersion for DIV is lower but still significant at the 10 percent level. We repeat the same regression by adding turnover as an additional independent variable and find that turnover is still the most significant explanatory variable and that the coefficient of dispersion is no longer significant. This indicates that turnover drives out the effect of dispersion on DIV.

4.4. Cross-sectional distribution of divergence of opinion over time

Before we investigate the cross-sectional relationship between divergence of opinion and stock returns, it is also interesting to examine the time series patterns of cross-sectional distribution of divergence of opinion.

Figure 1 plots the monthly cross-sectional average divergence of opinion over time (graybars denote the NBER recession dates and 1987 stock market crash). This measure, which is also defined as aggregate divergence of opinion, displays large time variation. Occasional upward spikes in the figure indicate months in which a stock market crash or economic recession occurred. The largest upward spike occurred in the month the stock market crashed, October 1987. The market-wide aggregate liquidity measure in Pastor and Stambaugh (2003) is lowest in the same period, indicating that when divergence of opinion and stock volatility is high, as in 1987 crash, compensa-
tions required by the liquidity provider are greater. The third largest upward spike is in November 1973, the first full month of the Middle East oil embargo and the subsequent 1973-1974 stock market crash. However, there are some upward spikes that occurred during economic booms, such as in February 1976. Therefore, aggregate divergence of opinion does not always coincide with low liquidity and high volatility. The correlation between the aggregate divergence of opinion and the aggregate liquidity measure in Pastor and Stambaugh (2003) is -0.22, confirming the insight obtained from Figure 1.

It will be interesting to further analyze the relationship between aggregate divergence of opinion and stock returns. As the main focus of this paper is to examine the effects of divergence of opinion on cross-sectional returns and the crucial role of short-sale constraints in determining the effects, we will leave that analysis to future work.

Figure 2 plots the 25th, 50th, and 75th percentiles for each month in the sample period for the cross-sectional distribution of DIV. All three percentiles appear to have similar time-varying patterns, with upward spikes in stock market crash and business recessions. The median, the 50th percentile, of the divergence of opinion is around 0.9. The reasonable time variations for all the percentiles and their nearly parallel movements over time show that the upward and downward spikes that we see in Figure 1 are not related to the unstable behavior of certain stocks, but rather to stock market characteristics.

5. Empirical results

The main objective of this paper is to examine the role of divergence of opinion in affecting future stock returns. In this section, we empirically investigate how the di-
vergence of opinion affect returns for different stocks with different arbitrage costs. We first sort stocks into five portfolios based on divergence of opinion of the previous month and hold the portfolios for one month. We calculate the equal-weighted monthly average returns of all the stocks in each of the five portfolios. Fama and French (1996) argue that the three-factor model in Fama and French (1993) captures many of the return anomalies. In the three-factor model, $R_m$ is the excess return on the market, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate, SMB (Small Minus Big) is the difference between the average return on three small portfolios and the average return on three big portfolios, and HML (High Minus Low) is the difference between the average return on two value portfolios and the average return on two growth portfolios. Following Carhart (1997), we include the momentum variable UMD into the three-factor model. UMD is the difference between past winners with high returns from $t-12$ to $t-2$ and the past losers with low returns from $t-12$ to $t-2$. We estimate the following four-factor model for each of the five portfolios sorted based on DIV:

$$E(R_{it} - R_{Ft}) = a_iR_m + s_iSML_t + b_iHML_t + m_iUMD_t.$$ (13)

The estimates of the intercepts in equation (13) are reported in the first column of table 2. The results clearly show that the lower the divergence of opinion, the higher the subsequent risk-adjusted return. When we move from the lowest divergence decile to

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12 See Fama and French (1996) for more details about how to construct these factors.
13 The results for the Fama-French three-factor model are similar; therefore, we only provide estimates of the four-factor model in this paper.
the highest divergence decile, the average monthly risk-adjusted return of the decile falls from 4.32 percent to -3.12 percent annually. The return difference between low- and high- divergence deciles is 7.44 percent annually and is statistically significant. This shows that for all stocks taken together, the results support Miller’s prediction.

To test our hypothesis and see if Miller’s prediction holds for stocks that are more difficult to arbitrage, we sort them based on the proxies for their arbitrage costs, which are short-sale costs, other transaction costs, and idiosyncratic risk.

5.1. The short-sale cost effect

We first look at the effects of short-sale cost on return differences between low- and high-divergence stocks. The short-interest-to-institutional-ownership ratio (SI/IO), which measures the difficulties involved in short selling, is our proxy for short sale costs. Each month, we assign stocks into five size quintiles based on the short-interest-to-institutional-ownership ratio (SI/IO) as of the end of the previous month. Then, within each SI/IO group, we further assign stocks into five quintiles (D1-D5) based on opinion divergence as of the end of the previous month. After assigning stocks into portfolios, stocks are held for one month. We calculate the equal-weighted monthly average returns of all the stocks in each portfolio and then estimate equation (13) for each of the twenty-five quintiles.

Table 2 presents the risk-adjusted returns for the two-way cut based on short sale cost and DIV. It shows that there is a strong negative relationship between opinion divergence and risk-adjusted returns for high-SI/IO and mid-SI/IO stocks. The four-factor risk adjusted returns of the portfolio decrease as we move from the low-divergence
group to the high-divergence group for those high-SI/IO stocks, which are more difficult to short sell. The effect is not as pronounced among low-SI/IO stocks. The return differential between low- and high-divergence groups is largest and significantly positive for the highest SI/IO group. Specifically, the annual return difference is 11.4 percent for the highest SI/IO group. However, the return difference is not significant and has the wrong sign for low-SI/IO and mid-SI/IO stocks. Because high-SI/IO stocks are more difficult to short, this result supports our hypothesis that, when opinions diverge, there are more likely to be arbitrage costs, lower future returns, and a bigger the return difference between low- and high-divergence stock groups.

5.2. Other transaction cost effect

In this subsection, we study how other transaction costs besides short-sale costs affect the divergence-return relationship. The proxies for the other transaction costs include firm size, price, dollar volume, frequency of zero return days and Amihud’s (2002) illiquidity measure. As before, we first assign stocks into five quintile groups(V1-V5) according to their proxies for transaction costs, then given any proxy within each of the five groups, we further sort stocks into five DIV groups(D1-D5) according to DIV. The four-factor risk-adjusted return differences between D1 and D5 across the five transaction cost measure deciles for each proxy and their t-statistics are reported in Table 3. The first row reports the risk-adjusted D1-D5 return differences for each of the five illiquidity-based groups V1-V5. The return differences between low- and high-divergence group for the three highest-illiquidity groups are 12 percent, 11.5 percent and 7 percent annually respectively. For lowest-illiquidity stocks, the number is
insignificant. We repeat the above analysis for each of our other transaction cost measures. The results show that D1-D5 return differences are significantly positive for low-price and small-size and high-zerofreq groups, which indicate high arbitrage costs. For example, the D1-D5 annual return difference is 19 percent for the lowest-price group, 18 percent for the smallest-size group and 15 percent for the highest-zerofreq groups. On the other hand, for high-price, large-size, and low-zerofreq group, the D1-D5 return differences are either insignificant or have the wrong sign. We did not see significantly positive return differences for low-volume groups.

5.3. Idiosyncratic risk effect

We use the standard deviation of the residuals for the four-factor model as a proxy for idiosyncratic risk (IR) and perform the same two-way curt as before based on IR and DIV. Each month we rank firms based on IR, and assign firms to a quintile based on those rankings. Within each IR group, we further assign stocks to five DIV groups.

Risk-adjusted returns of the twenty-five portfolios are reported in Table 4. The annual return differentials between low- and high-divergence stocks for the high-IR group and mid-IR group are significantly positive, with magnitudes of 26 percent, 10 percent and 4 percent, respectively. This provides further corroboration that divergence of opinion causes overpricing in stocks that are more difficult and costly to arbitrage.

5.4. Cross-sectional regressions

To examine the role that different types of arbitrage costs play in determining the divergence of opinion’s effect on stock returns, we run the Fama-MacBeth cross-sectional
regression of returns on the arbitrage cost measures and the interaction of DIV and those arbitrage cost measures, controlling for anomalies documented in the literature.

Table 5 reports the coefficients and Newey-West adjusted t-statistics of the regressions. We did not include the proxy for short sale costs in the regressions over the period 1976 to 2003, because the sample of short interest starts in 1991. Before we add interaction items, our measure of divergence of opinion has negative effects on returns with a significant level of -5.62 in regression (1). This is presented in the first column in Table 5 and is also consistent with the portfolio results in Table 2. In regression (2), we run the regression of returns on the idiosyncratic risk measure and the interaction of DIV and the idiosyncratic risk measure, controlling for beta, size, $BE/ME$ and momentum effects. The significant negative coefficient on the interaction term $DIV \times IR$ indicates that when idiosyncratic risk is high, the DIV effect on returns also is high, which is consistent with our portfolio analysis in Table 4. The third column presents the results of regression (3), which is the regression of the complete arbitrage cost variables and the interactions of DIV and these variables. The coefficient on the interaction of $DIV \times IR$ is still significantly negative, while the coefficients on other interaction terms are not significant. This suggests that idiosyncratic risk has important explanatory power on the divergence of opinion effect. We also repeat regression (3) over period 1991 to 2003 by adding the short-sale cost measure (SI/IO) and the interaction of DIV and SI/IO although the result of is not presented in Table 5. After controlling for short-sale cost and its interaction with DIV, we still find a negative significant coefficient (-3.006, t-stat=-3.664) on $DIV \times IR$, which confirms the important role played
by idiosyncratic risk on explaining the DIV effect.

5.5. Robustness check

In this section, we demonstrate that the overpricing caused by divergence of opinion given short-sale constraints is robust to additional trading strategies.

5.5.1. Subperiod analysis

We divide the sample into three sub-periods: 1970-1982, 1983-1992, and 1993-2003. Table 6 reports the mean monthly return differentials between low- and high-divergence groups for each of five quintiles based on three arbitrage cost measures: idiosyncratic risk, illiquidity, and price. Clearly there is a declining trend for all the three variables. For idiosyncratic risk, the return differences are significantly positive for the three highest-IR quintiles for the period 1970-1982, with respective magnitudes of 2.59, 1.26, and 1.14. The return differential is also significantly positive for the highest IR-groups, with magnitude of 2.28 for the period 1983-1992, while for the period 1993-2003, the significantly positive return differentials are concentrated for the two smallest IR-group. Moreover, within each IR-groups, the magnitudes of the return differentials are lower for recent periods. Specifically, they move down from 2.59 to 1.69 percent when we come from the 1970s to recent periods for highest-IR group, 1.26 to 0.96 for the second-highest group, and 1.14 to -0.2 for the mid-IR group. A similar trend can also be seen for the subperiod analysis for illiquidity and price. However, even for the most recent period, the results still confirm our previous conclusion that the negative effect of divergence of opinion is most pronounced for stocks with higher
arbitrage costs.

5.5.2. Different lags in portfolio formation

To see how long the overpricing for all stocks and different size deciles can last, we assign stocks to portfolios after a wait of several months. Figure 2 presents the mean return differentials between low- and high-divergence groups after waiting for zero to eleven months for all the stocks and for stocks in the five size-groups. S1 denotes the smallest-size group, and S5 the largest. The figure shows that, return differentials for all stocks and all size groups decline as the lag becomes longer. Overpricing disappears because, as more months pass before we form the portfolio, investors are better informed about the stock’s true value, while optimistic investors are becoming disappointed and selling the stock. The return differential becomes insignificant if the lag is longer than eight months for all stocks, ten for S1, nine for S2, six for S3, and three for S4. The smaller the stock’s market capitalization, the longer it takes for overpricing to disappear. This result, again, is consistent with our hypothesis. These results also shed light on the strategy of investors: As the overpricing caused by divergence of opinion takes several months to disappear, and as it is most difficult or costly to take a short position in those small stocks for which overpricing is most pronounced, it is better to avoid buying overvalued stocks bid up by overconfident investors.

5.5.3. Discussion

The basic model of Tauchen and Pitts (1983) doesn’t consider the effect of limits of arbitrage on the stock price, volume and $\sigma^2$, our proxy for divergence of opinion. When
arbitrage costs prevent some short sellers from taking the short positions, our proxy \( \sigma^2 \) captures the differences of opinion among all the \( M \) (\( M < J \)) active traders, rather than among all the \( J \) traders when there are no arbitrage costs. Therefore, \( \sigma^2 \) might underestimate the true divergence of opinion among all investors. However, it won’t change our empirical results, because we are interested in how divergence of opinion affects stocks returns for stocks with different arbitrage costs. For stocks with smallest arbitrage costs, the downward bias in the \( \sigma^2 \) estimation is negligible. Even for stocks within the largest arbitrage costs portfolio, \( \sigma^2 \) underestimates the true divergence of opinion for all the stocks to a similar degree in that group. The higher the estimate \( \sigma^2 \) is, the higher the true divergence of opinion among all investors are for the stocks within the same arbitrage cost group, given arbitrage costs. Therefore, our results that divergence of opinion leads to upwardly biased prices in the presence of arbitrage costs are still valid.

6. Conclusion

In this paper we provide a new proxy for divergence of opinion among all individual investors, estimated from the generalized model of Tauchen and Pitts (1983). Our measure of divergence of opinion has significant relations with other proxies for divergence of opinion, such as dispersion in analysts’ earnings forecast and turnover. As a proxy for divergence of opinion, our measure has several advantages compared to those in the literature to date. It is isolated from the joint distribution of the volatility and volume and can directly capture the information of divergence of opinion among all individual investors, as opposed to dispersion in analysts’ earnings forecasts (which is only a
proxy for differences of opinion among analysts) and turnover (which is also a proxy for other variables).

We use our measure DIV to examine the cross-sectional effects of divergence of opinion on future stock returns. Our results shows that divergence of opinion results in upwardly biased prices when arbitrage costs are high. We sort stocks into different classes according to their proxies for arbitrage costs such as idiosyncratic risk, short-sale cost and other transaction costs and find that stock prices are mainly upwardly biased, and that return differences between low-divergence and high-divergence stock groups are biggest for small-size, low-price, high-illiquidity, high-short-sale-cost, high-zerofreq, and high-idiosyncratic-risk stocks. This is consistent with Miller’s theory, while the Hong and Stein (2003) model’s prediction of unbiased prices is valid for stocks with lower arbitrage costs. Our results also suggest that idiosyncratic risk plays a more important role in the existence of divergence of opinion effect than do other types of arbitrage costs.
Appendix

A.1. Estimation Method: Simulated MLE

Let \( y_t = \{ r_t, v_t \}^T_{t=1} \) denote the matrix of observable variables \( r_t \) and \( v_t \), \( x_t = \{ \eta_t \}^T_{t=1} \) denote the vector of latent variables \( \eta_t \) and \( f(y_t, x_t|\theta) \) represent the joint density of \( x_t \) and \( y_t \), where \( \theta \) denotes the vector of parameters to be estimated. The likelihood function of generalized model in section 1.2 associated with the observable variables is given by the following integral

\[
L(\theta; y_t) = \int_{\omega} f(y_t, x_t|\theta) dx_t,
\]  
(14)

where \( \omega \) is the support over \( R^{2T} \) for models with the dynamic latent variable. In the SML approach, the likelihood (14) is evaluated by a MC technique based on an importance sampling procedure and then maximized to obtain estimates of \( \theta \).

We can factorize \( f(y_t, x_t|\theta) \) into \( k(y_t|x_t, \theta) = \prod_{t=1}^{T} k(y_t|x_t, \theta) \) and \( g(x_t|\theta) = \prod_{t=1}^{T} g(x_t|x_{t-1}, \theta) \), where \( k(y_t|x_t, \theta) \) denotes the joint density of observable variables return \( r_t \) and trading volume \( v_t \) conditional on latent variables \( x_t = \{ \eta_t \} \), and \( g(x_t|x_{t-1}, \theta) \) denotes the conditional density of the latent variables given their past values. Then equation (14) can be written as

\[
L(\theta; y_t) = \int_{\omega} \prod_{t=1}^{T} k(y_t|x_t, \theta)g(x_t|\theta) dx_t = \int_{\omega} \prod_{t=1}^{T} k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta)dx_t. 
\]  
(15)

We know that \( k(y_t|x_t, \theta) \) is bivariate Gaussian and \( g(x_t|x_{t-1}, \theta) \) is univariate Gaussian, so if we regard (15) as expected value of \( k(y_t|x_t, \theta) \) on the distribution \( g(x_t|\theta) \)
and simulate $N$ i.i.d. samplers $\{\tilde{x}_{t,n}\}_{n=1}^{N}$ drawn from the distribution $g(x_t|\theta)$, which we refer to as natural samplers, we can easily obtain a natural MC estimator of $L(\theta; y_t)$ as following:

$$\frac{1}{N} \sum_{n=1}^{N} k(y_t|\tilde{x}_{t,n}, \theta). \quad (16)$$

However, this MC estimate is based on a sequence of sampling densities $g$ which do not take into account the fact that the observations of $r_t$ and $v_t$ contain critical information on the underlying latent process. It is shown by Danielsson and Richard (1993) that a prohibitively huge simulation sample size $N$ would be required to obtain an accurate natural MC estimator and the natural MC estimator is highly inefficient. In order to address the inefficiency problem, we follow Liesenfeld and Richard (2001) to apply Efficient Importance Sampling (EIS) that significantly generalized Accelerated Gaussian Importance Sampling (AGIS) procedure developed by Danielsson and Richard (1993). EIS searches for a sequence of samplers that exploits the information on the $\eta_t$'s conveyed by $r_t$ and $v_t$. This procedure is to find an auxiliary Gaussian sampler $s(x_t|\gamma)$ indexed by the auxiliary parameter vector $\gamma$ that can minimize the MC sampling variance of the corresponding MC estimator of $L(\theta; y_t)$. For any given value of $\gamma$, using $s(x_t|\gamma)$ equation (15) can be rewritten as

$$L(\theta; y_t) = \int \frac{k(y_t|x_t, \theta)g(x_t|\theta)}{s(x_t|\gamma)} s(x_t|\gamma) dx_t, \quad (17)$$

If we can draw $N$ i.i.d. trajectories from the sampler $s(x_t|\gamma)$, which is referred as importance sampler, then the corresponding MC estimator is
\[ \hat{L}_N(\theta; y_t) = \frac{1}{N} \sum_{n=1}^{N} \frac{k(y_t|\tilde{x}_{t,n}, \theta)g(\tilde{x}_{t,n}|\theta)}{s(\tilde{x}_{t,n}|\gamma)}. \] (18)

It follows that the MC sampling variance of importance MC estimator (18) is

\[ Var[\hat{L}_N(\theta; y_t)] = \frac{1}{N} Var_s[\frac{k(y_t|x_t, \theta)g(x_t|\theta)}{s(x_t|\gamma)}]. \] (19)

Note that if the value of the parameter vector \( \gamma \) could be found such that the density \( s(x_t|\gamma) \) is proportional to \( k(y_t|x_t, \theta)g(x_t|\theta) \), then MC sampling variance of the importance MC estimator would be zero. EIS is used to search the parameter vector \( \gamma \) which can make the shape of \( s(x_t|\gamma) \) match that of \( k(y_t|x_t, \theta)g(x_t|\theta) \) as well as possible so that the MC sampling variance of \( \hat{L}_N(\theta; y_t) \) can be minimized. However, this is a high-dimensional integration problem, so it is computationally infeasible to solve the least squares optimization problem. It is necessary to break down the high-dimensional optimization problem into a sequence of separate low-dimensional optimization problems for each period \( t \) as suggested by the factorization in (15) and the factorization of \( s(x_t|\gamma) \) into \( \prod_{t=1}^{T} s(x_t|x_{t-1}, \gamma) \). Because the integral of \( k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta) \) with respect to \( x_t \) depends on \( x_{t-1} \) and the integral of \( s(x_t|x_{t-1}, \gamma) \) is by definition equal to one, it is impossible to obtain a good match between \( k.g \) and \( s \) period by period independently. However, we could find a positive functional approximation \( p(x_t; a_t) \) for the density \( k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta) \), which can be analytically integrable with respect to \( \eta_t \). And \( s(x_t|x_{t-1}, \gamma) \) can be written as

\[ s(x_t|x_{t-1}, \gamma) = \frac{p(x_t; \gamma)}{\chi(x_{t-1}, \gamma)}, \text{ where } \chi(x_{t-1}, \gamma) = \int p(x_t; \gamma) d\eta_t. \] (20)
It follows that we need to find a class of density kernel \( p \) for the auxiliary importance samplers \( s \) so that \( s \) can be a good functional approximation for \( k.g \). A natural choice for \( s \) is to use parametric extension the natural samplers \( g \), therefore the following parametrization for the density kernel \( p \) is suggested

\[
p(x_t; \gamma_t) = g(\eta_t|\eta_{t-1}, \theta)\xi(\eta_t, \gamma_t),
\]

where the auxiliary function \( \xi(\eta_t, \gamma_t) \) is a Gaussian density kernel and \( \xi(\eta_t, \gamma_t) = \exp(\gamma_0 + \gamma_1 \eta_t + \gamma_2 \eta_t^2) \). Note that \( p \), which is the multiplication of two density kernels of Gaussian distribution, is closed.

Equation (21) also implies that \( p(x_t; \gamma_t) \) is a Gaussian density kernel for \( \eta_t \) given \( \eta_{t-1} \). If there is a good match between \( k(y_t|x_t, \theta)g(x_t|x_{t-1}, \theta) \) and \( p(x_t; \gamma_t) \), then \( \chi(x_{t-1}, \gamma_t) \) will be unaccounted for. Nevertheless, as \( \chi(x_{t-1}, \gamma_t) \) does not depend on \( \eta_t \), it can be transferred back to the minimization problem for period \( t-1 \). Therefore, the following back-recursive sequence of least-squares problems need to be solved:

\[
\hat{\gamma}_t = \arg\min_{\gamma_t} \sum_{i=1}^{N} \left\{ \ln[k(y_i|x_i, \theta)g(x_i|x_{i-1}, \theta)\chi(x_i, \gamma_{t+1})] - \ln p(x_i; \gamma_t) \right\}^2
\]

Note that as the density kernel \( p \) is chosen within the exponential family of distributions as in (21), the least-squares problem in equation (22) is linear in \( \gamma_t \). After we obtain \( \hat{\gamma}_t \), we can estimate the likelihood function (17) for any given value \( \theta \).
A.2. The implementation of EIS for the generalized model

The generalized model characterized by equation (11) assumes return $r_t$ and volume $v_t$ given $\eta_t$ and $\eta_t$ given its past value $\eta_{t-1}$ are Gaussian distributed. Their densities are given by:

\begin{align}
\kappa(y_t|x_t, \theta) &\propto \exp\left\{-\frac{r_t^2}{2\sigma^2_{rt}}\right\}\exp\left\{-\frac{(v_t - \mu_{vt})^2}{2\sigma^2_{vt}}\right\} \\
g(x_t|\theta) &\propto \exp\left\{-\frac{(\eta_t - \delta_{\eta} \eta_{t-1})^2}{2\sigma^2_{\eta}}\right\}
\end{align}

where

\begin{align*}
\sigma^2_{rt} &= \beta_1 + \beta_2 e^{\eta_t}, \\
\mu_{vt} &= \beta_3 e^{\eta_t/2},
\end{align*}

and

\[\sigma^2_{vt} = \beta_4 e^{\eta_t}.\]

The multiplicative factors which do not depend upon $\eta_t$ are omitted.

Therefore, according to equation (21), the density kernels of the importance samplers have the form

\[p(x_t; \gamma) \propto \exp\left\{-\frac{1}{2}\left[\frac{1}{\sigma^2_{\eta}} - 2\gamma_{2,t}\right] \eta_t^2 - 2\left(\frac{\delta_{\eta} \eta_{t-1}}{\sigma^2_{\eta}} + \gamma_{1,t}\right) \eta_t + \left(\frac{\delta_{\eta} \eta_{t-1}}{\sigma_{\eta}}\right)^2\right\}\]

Accordingly, the conditional mean and variance of $\eta_t$ on the density kernel $s(x_t|\gamma)$ is the following

34
\[ \mu_{s,t} = \sigma_{s,t}^2 \left( \frac{\delta_{\eta_{t-1}}}{\sigma_\eta^2} + \gamma_{1,t} \right), \quad \sigma_{s,t}^2 = \frac{\sigma_\eta^2}{1 - 2\sigma_\eta^2 a_{2,t}}, \] (26)

After integrating \( p(x_t; \gamma) \) with respect to \( \eta_t \), we have the following expression for the integrating constant:

\[ \chi(x_{t-1}, \gamma) \propto \exp \left\{ \frac{\mu_{s,t}^2}{2\sigma_{s,t}^2} - \frac{(\delta_{\eta_{t-1}})^2}{2\sigma_\eta^2} \right\} \] (27)

To implement EIS, the following procedure is required:

Step 1: Draw \( N \) trajectories of the latent variable \( x_t = \{ \eta_t \} \) from the natural sampler \( g \).

Step 2: Solve the back-recursive sequence of least-squares problems defined in (22) to obtain the solutions \( \{ \tilde{\gamma}^*_{t} \} \) using these random draws. For every period \( t = 1, ..., T \), based on \( N \) observations run the following linear auxiliary regression:

\[ \ln k_t(y_t | \tilde{\eta}_{t,n}, \theta) + \ln (\chi(\tilde{\eta}_{t,n}, \gamma_{t+1})) = \gamma_{0,t} + \gamma_{1,t} \tilde{\eta}_{t,n} + \gamma_{2,t} \tilde{\eta}_{t,n}^2 + \text{residual} \] (28)

Step 3: Based on \( \{ \tilde{\gamma}^*_{t} \} \), determine the sequence samplers \( s(x_t | \gamma^*_{t}) \) which have conditional mean and conditional variance given in equation (26). Use these new sequence samplers to draw \( N \) new simulated sample \( \{ \tilde{\eta}_{t,n} \}_{n=1}^{N} \).

Step 4: A small number of iterations of step 2 and step 3 are required to obtain the efficient importance samplers.

Step 5: The likelihood function characterized by equation (18) is estimated using the new simulated sample \( \{ \tilde{\eta}_{t,n} \}_{n=1}^{N} \) and estimators for \( \gamma \) are obtained.
More details about implementing EIS to search for $s(x_t|\gamma)$ and estimating the mixture model are provided by Danielsson and Richard (1993) and Liesenfeld (2001).
References


Table 1. Fama-MacBeth Regressions of Divergence of Opinion on Lagged Firm Characteristics

Fama and MacBeth (1973) cross-sectional regressions are run every month from February 1983 to Dec 2003. The dependent variable is our measure of monthly divergence of opinion (DIV). Beta is estimated using the past 60 months of data, lnME is the log of one month lagged ME, lnB/M is the log of one month lag of BE/ME, Disp is dispersion in analysts’ earnings forecasts, Turn is turnover, PIN is probability of information-based trading proposed in Easley, Kiefer, O’Hara and Paperman (1996). *, **, or *** indicate statistical significance at 1%, 5% and 10% respectively.

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<td>(5.349)</td>
<td>(3.757)</td>
<td>(-2.645)</td>
<td>(1.791)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.045***</td>
<td>0.011***</td>
<td>0.057***</td>
<td></td>
<td></td>
<td>0.439***</td>
</tr>
<tr>
<td>(-9.003)</td>
<td>(2.412)</td>
<td>(13.829)</td>
<td></td>
<td></td>
<td>(26.029)</td>
</tr>
<tr>
<td>-0.098***</td>
<td>0.044***</td>
<td>0.062***</td>
<td>0.008</td>
<td>0.440***</td>
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</tr>
<tr>
<td>(-10.33)</td>
<td>(8.45)</td>
<td>(11.78)</td>
<td>(1.173)</td>
<td></td>
<td>(23.11)</td>
</tr>
</tbody>
</table>
Table 2. Risk-Adjusted Returns of Portfolios Based on SI/IO and DIV The table presents risk-adjusted returns (relative to a four-factor model $E(R_{it} - R_{ft}) = a_i R_{Mt} + s_i SMB_t + b_i HML_t + m_i UMD_t$) for portfolios based on short-interest-to-institutional-ownership ratio (SI/IO) and DIV. For each month $t$, stocks are sorted into five size groups based on SI/IO at the end of previous month. Stocks in each SI/IO group are then sorted into five additional groups based on DIV at the end of the previous month. Stocks are held for one month and the portfolio returns are equal-weighted. The short-interest-to-institutional-ownership ratio (SI/IO) is obtained by dividing short interest by the institutional ownership. The market premium uses the CRSP NYSE/AMEX/Nasdaq value-weighted index. The variables HML and SMB are created using the same methodology as Fama and French (1996). The momentum premium (UMD) is the difference between the return on a portfolio comprised of stocks with high returns and the return on a portfolio comprised of stocks with low returns from $t - 12$ to $t - 2$. The table reports alphas of the regressions over the period February 1970 to December 2003. ‘∗’, ‘∗∗’, or ‘∗∗∗’ indicate statistical significance at 1%, 5% and 10% respectively.

<table>
<thead>
<tr>
<th>DIV Quintiles</th>
<th>Short-Interest-to-Institutional-Ownership Ratio Quintiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
</tr>
<tr>
<td>D1(low)</td>
<td>0.36***</td>
</tr>
<tr>
<td></td>
<td>(4.10)</td>
</tr>
<tr>
<td>D2</td>
<td>0.15***</td>
</tr>
<tr>
<td></td>
<td>(2.09)</td>
</tr>
<tr>
<td>D3</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.89)</td>
</tr>
<tr>
<td>D4</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>D5(high)</td>
<td>-0.26***</td>
</tr>
<tr>
<td></td>
<td>(-3.39)</td>
</tr>
<tr>
<td>D1-D5</td>
<td>0.62***</td>
</tr>
<tr>
<td></td>
<td>(5.16)</td>
</tr>
</tbody>
</table>
Table 3. Risk-Adjusted Returns of Portfolios Based on Transaction Costs and DIV

The table presents risk-adjusted returns (relative to a four-factor model $E(R_{it} - R_{ft}) = a_i R_M + s_i SMB_t + b_i HML_t + m_i UMD_t$ for D1-D5 portfolios based on DIV, across quintiles of the proxies for other transaction costs, which are illiquidity, price, volume, zero freq and ME. The equal-weighted portfolios sorted based on each proxy, and DIV are formed the same way as in Table 2. The risk-adjusted return difference between the low and high DIV quintiles for the five deciles of each of five transaction cost measures are computed over the period February 1970 to December 2003. Illiquidity is the average of the ratio of daily absolute return to daily volume from $t - 13$ to $t - 2$. Price is the closing price of a share of common stock at the end of month $t - 1$. Volume is the monthly average of the daily volume of the stock at month $t - 1$. Zero freq is the number of days with zero returns over the past 12 months. ME is the market value of equity in millions of dollars at the end of month $t - 1$. The market premium uses the CRSP NYSE/AMEX/Nasdaq value-weighted index. The variables HML and SMB are created using the same methodology as Fama and French (1996). The momentum premium (UMD) is the difference between the return on a portfolio comprised of stocks with high returns and the return on a portfolio comprised of stocks with low returns from $t - 12$ to $t - 2$. “∗”, “∗∗”, or “∗∗∗” indicate statistical significance at 1%, 5% and 10% respectively.
<table>
<thead>
<tr>
<th>Transaction Cost Quintiles</th>
<th>V1 (Small)</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5 (Large)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illiquidity</td>
<td>0.20</td>
<td>0.36**</td>
<td>0.59***</td>
<td>0.96***</td>
<td>1.03***</td>
</tr>
<tr>
<td></td>
<td>(1.35)</td>
<td>(2.24)</td>
<td>(3.50)</td>
<td>(5.86)</td>
<td>(5.79)</td>
</tr>
<tr>
<td>Price</td>
<td>1.60***</td>
<td>1.03***</td>
<td>0.57***</td>
<td>-0.01</td>
<td>-0.32**</td>
</tr>
<tr>
<td></td>
<td>(8.29)</td>
<td>(5.80)</td>
<td>(3.62)</td>
<td>(-0.07)</td>
<td>(-2.20)</td>
</tr>
<tr>
<td>Volume</td>
<td>0.41***</td>
<td>0.84***</td>
<td>0.74***</td>
<td>0.80***</td>
<td>0.48***</td>
</tr>
<tr>
<td></td>
<td>(2.93)</td>
<td>(5.87)</td>
<td>(4.89)</td>
<td>(4.95)</td>
<td>(2.67)</td>
</tr>
<tr>
<td>Zerofreq</td>
<td>0.22</td>
<td>0.44***</td>
<td>0.63***</td>
<td>0.88***</td>
<td>1.27***</td>
</tr>
<tr>
<td></td>
<td>(1.42)</td>
<td>(2.72)</td>
<td>(3.79)</td>
<td>(5.06)</td>
<td>(7.33)</td>
</tr>
<tr>
<td>ME</td>
<td>1.54***</td>
<td>1.17***</td>
<td>0.56***</td>
<td>0.12</td>
<td>-0.43***</td>
</tr>
<tr>
<td></td>
<td>(8.81)</td>
<td>(6.56)</td>
<td>(3.00)</td>
<td>(0.78)</td>
<td>(-3.50)</td>
</tr>
</tbody>
</table>
Table 4. Risk-Adjusted Returns of Portfolios Based on Idiosyncratic Risk and DIV

The table presents risk-adjusted returns (relative to a four-factor model $E(R_{it} - R_{ft}) = a_i R_{Mt} + s_i SMB_t + b_i HML_t + m_i UMD_t$) for portfolios based on idiosyncratic risk and DIV. The equal weighted portfolios sorted based on idiosyncratic risk (IR) and DIV are formed the same way as in Table 2. IR is the standard deviation of the residuals from the four-factor model for the past 60 months. The market premium uses the CRSP NYSE/AMEX/Nasdaq value-weighted index. The variables HML and SMB are created using the same methodology as Fama and French (1996). The momentum premium (UMD) is the difference between the return on a portfolio comprised of stocks with high returns and the return on a portfolio comprised of stocks with low returns from $t - 12$ to $t - 2$. The table reports alphas of the regressions over period February 1970 to December 2003. "∗"", "∗∗", or "∗∗∗" indicate statistical significance at 1%, 5% and 10% respectively.

<table>
<thead>
<tr>
<th>DIV Quintiles</th>
<th>IR1 (Small)</th>
<th>IR2</th>
<th>IR3</th>
<th>IR4</th>
<th>IR5 (Large)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1 (low)</td>
<td>0.04</td>
<td>0.07</td>
<td>0.34***</td>
<td>0.51***</td>
<td>0.92***</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.60)</td>
<td>(2.85)</td>
<td>(3.58)</td>
<td>(3.06)</td>
</tr>
<tr>
<td>D2</td>
<td>0.06</td>
<td>0.12</td>
<td>0.22**</td>
<td>0.21**</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(1.12)</td>
<td>(2.03)</td>
<td>(1.65)</td>
<td>(0.56)</td>
</tr>
<tr>
<td>D3</td>
<td>0.19***</td>
<td>0.21**</td>
<td>0.13</td>
<td>0.06</td>
<td>-0.24</td>
</tr>
<tr>
<td></td>
<td>(1.99)</td>
<td>(2.04)</td>
<td>(1.19)</td>
<td>(0.46)</td>
<td>(-1.63)</td>
</tr>
<tr>
<td>D4</td>
<td>0.26***</td>
<td>0.22**</td>
<td>0.10</td>
<td>-0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(2.75)</td>
<td>(2.25)</td>
<td>(0.94)</td>
<td>(-1.26)</td>
<td>(-1.10)</td>
</tr>
<tr>
<td>D5 (high)</td>
<td>0.41***</td>
<td>0.31***</td>
<td>0.01</td>
<td>-0.30**</td>
<td>-1.21***</td>
</tr>
<tr>
<td></td>
<td>(4.47)</td>
<td>(3.26)</td>
<td>(0.10)</td>
<td>(-2.23)</td>
<td>(-7.09)</td>
</tr>
<tr>
<td>D1-D5</td>
<td>-0.38***</td>
<td>-0.24**</td>
<td>0.33**</td>
<td>0.80***</td>
<td>2.13***</td>
</tr>
<tr>
<td></td>
<td>(-3.92)</td>
<td>(-1.98)</td>
<td>(2.17)</td>
<td>(4.02)</td>
<td>(9.80)</td>
</tr>
</tbody>
</table>

Fama and MacBeth (1973) cross-sectional regressions are run every month from February 1976 to December 2003. The dependent variable is expected returns of all the individual stocks in the market. The independent variables are listed in the first column of the table. Beta is estimated following Fama and French (1992) using the past 60 months of data, ln(ME) is the log of ME at \( t - 1 \), ln(B/M) is the log of BE/ME at \( t - 1 \), is the past return at month \( t - 1 \), is the past return from \( t - 12 \) to \( t - 2 \), and is the past return from \( t - 36 \) to \( t - 13 \). DIV is our proxy for divergence of opinion. The table reports coefficients of the independent variables over the period February 1976 to December 2003. *, **, and *** indicate statistical significance at 1%, 5% and 10% respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>0.001</td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>ln(ME)</td>
<td>-0.005</td>
<td>-0.09***</td>
<td>-0.32***</td>
</tr>
<tr>
<td>ln(B/M)</td>
<td>0.23***</td>
<td>0.18***</td>
<td>0.14**</td>
</tr>
<tr>
<td>Shortret</td>
<td>0.13***</td>
<td>0.14***</td>
<td>0.15***</td>
</tr>
<tr>
<td>Lret</td>
<td>-0.03***</td>
<td>-0.025***</td>
<td>-0.03***</td>
</tr>
<tr>
<td>Longret</td>
<td>-0.07***</td>
<td>-0.059***</td>
<td>-0.04***</td>
</tr>
<tr>
<td>DIV</td>
<td>-0.10***</td>
<td>0.30***</td>
<td>-0.07</td>
</tr>
<tr>
<td>DIV × IR</td>
<td>-3.51***</td>
<td>-3.32***</td>
<td></td>
</tr>
<tr>
<td>DIV × Price</td>
<td>0.003**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIV × ln(Volume)</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIV × ln(ME)</td>
<td>-0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIV × Illiquidity</td>
<td>0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIV × Zerofreq</td>
<td>-0.0005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>-5.38***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Volume)</td>
<td>0.27***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Illiquidity</td>
<td>10.88***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zerofreq</td>
<td>0.005*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 6. Subperiod Analysis

The table presents the four-factor risk-adjusted return differential between stocks in the lowest- and highest- dispersion groups in each IR, Illiquidity, and Price group over the indicated periods. t-statistics are in parentheses. The equal weighted portfolios sorted based on each of the three variables, and DIV are formed the same way as in Table 2. The risk-adjusted return difference between the low and high DIV quintiles D1-D5 for the five deciles of each of the three variables are computed and reported in the table. Illiquidity is the average of the ratio of daily absolute return to daily volume from $t-13$ to $t-1$. Price is the closing price of a share of common stock at the end of month $t-1$. IR is the standard deviation of the residuals from the four-factor model for past 60 months. “∗”, “∗∗”, or “∗∗∗” indicate statistical significance at 1%, 5% and 10% respectively.
<table>
<thead>
<tr>
<th>Time Period</th>
<th>V1 (Small)</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5 (Large)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-1982</td>
<td>-0.26</td>
<td>0.35</td>
<td>1.14 ***</td>
<td>1.26 ***</td>
<td>2.59 ***</td>
</tr>
<tr>
<td></td>
<td>(-1.33)</td>
<td>(1.62)</td>
<td>(4.29)</td>
<td>(3.98)</td>
<td>(7.55)</td>
</tr>
<tr>
<td>1983-1992</td>
<td>-0.36 **</td>
<td>-0.15</td>
<td>0.11</td>
<td>0.28</td>
<td>2.28 ***</td>
</tr>
<tr>
<td></td>
<td>(-2.41)</td>
<td>(-0.82)</td>
<td>(0.53)</td>
<td>(1.20)</td>
<td>(7.67)</td>
</tr>
<tr>
<td>1993-2003</td>
<td>-0.52 ***</td>
<td>-0.78 ***</td>
<td>-0.2</td>
<td>0.96 **</td>
<td>1.69 ***</td>
</tr>
<tr>
<td></td>
<td>(-3.35)</td>
<td>(-3.71)</td>
<td>(-0.76)</td>
<td>(2.38)</td>
<td>(3.80)</td>
</tr>
<tr>
<td><strong>Illiquidity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-1982</td>
<td>0.59 **</td>
<td>0.38</td>
<td>0.92 ***</td>
<td>1.35 ***</td>
<td>1.82 ***</td>
</tr>
<tr>
<td></td>
<td>(2.28)</td>
<td>(1.52)</td>
<td>(3.51)</td>
<td>(4.54)</td>
<td>(5.74)</td>
</tr>
<tr>
<td>1983-1992</td>
<td>0.10</td>
<td>0.23</td>
<td>0.52 **</td>
<td>0.69 ***</td>
<td>0.63 ***</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(1.13)</td>
<td>(2.47)</td>
<td>(2.79)</td>
<td>(0.64)</td>
</tr>
<tr>
<td>1993-2003</td>
<td>0.16</td>
<td>0.52</td>
<td>0.34</td>
<td>0.84 ***</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.59)</td>
<td>(1.49)</td>
<td>(0.93)</td>
<td>(3.03)</td>
<td>(1.43)</td>
</tr>
<tr>
<td><strong>Price</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-1982</td>
<td>2.01 ***</td>
<td>1.47 ***</td>
<td>0.86 ***</td>
<td>0.42 *</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(6.38)</td>
<td>(5.05)</td>
<td>(3.37)</td>
<td>(1.88)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>1983-1992</td>
<td>1.43 ***</td>
<td>1.11 ***</td>
<td>0.59 ***</td>
<td>-0.29</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>(5.24)</td>
<td>(4.72)</td>
<td>(2.75)</td>
<td>(-1.42)</td>
<td>(-1.19)</td>
</tr>
<tr>
<td>1993-2003</td>
<td>1.28 ***</td>
<td>0.72 **</td>
<td>0.39</td>
<td>-0.06</td>
<td>-0.61 **</td>
</tr>
<tr>
<td></td>
<td>(3.27)</td>
<td>(2.00)</td>
<td>(1.23)</td>
<td>(-0.20)</td>
<td>(-2.09)</td>
</tr>
</tbody>
</table>
Figure 1. Cross-Sectional Average of Divergence of Opinion

The figure shows the cross-sectional average of the monthly divergence of opinion across all the stocks in the market over time. The sample period is from August 1962 to December 2003.
Figure 2. Cross-Sectional Distribution of Divergence of Opinion

The figure shows the 25th, 50th, and 75th percentiles each month for the cross-sectional distribution of divergence of opinion across all the stocks in the market over time. The sample period is from August 1962 to December 2003.
Figure 3. Return Differentials for Varying Lags in Portfolio Formation

Stocks are assigned to size deciles based on market capitalization at the end of the previous 1 month; within each size decile, we further assign stocks into groups based on divergence of opinion in the previous 1 month. Using various lag periods, we wait to assign stocks into equal-weighted portfolios and calculate differences in returns between low- and high-divergence of opinion groups. The sample period is February 1970 to December 2003.