Vertical Mergers in Procurement Markets*

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Abstract

This paper evaluates the profitability and welfare effects of vertical mergers in a duopoly procurement setting that incorporates both incomplete information about sellers’ costs, and asymmetries in sellers’ productive capabilities. The analysis requires a computational approach, and it reveals that vertical mergers frequently are jointly unprofitable, but that at least one of the two mergers available always is jointly profitable. The buyer’s most profitable merger partner almost always is the seller with lower expected costs, and is typically the larger seller. Finally, vertical mergers frequently reduce total welfare, although postmerger welfare tends to be higher the larger is the buyer’s merger partner. Some of the results contrast qualitatively with unambiguous findings from less general models, which suggests that caution be used in drawing inferences from those models.

1 Introduction

An important theme in the vast literature on vertical integration is evaluating the strategic effects of vertical mergers when there exists imperfect competition at one or more levels of the chain of production. Relevant issues for market participants and regulatory agencies include merger profitability, changes in the profits of unintegrated firms and potential entrants, and changes in total and consumer welfare. For example, one might consider under what conditions Dell and Intel could profitably integrate their operations, and the effects of such a change both on final consumers, and on rival computer or microchip manufacturers.

A standard modeling approach used to analyze these issues is to consider one or more upstream firms that sell inputs to one or more downstream firms, with at least one level characterized by imperfect competition. This approach typically assumes there exists complete information about demand and cost conditions, and that market participants at a given level are symmetrically positioned with respect to those demand and cost conditions. Generally speaking, the focus is on environments in which upstream and downstream firms participate in an intermediate goods market, the outcomes of which influence subsequent competition amongst the downstream firms in a final goods market. For example, a commonly employed strategic scenario consists of two upstream sellers S1 and S2 that sell an intermediate good to two downstream buyers B1 and

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1 See Perry [1989] for an overview of the vertical integration literature.

2 For example, see Salinger [1988], Ordover, Saloner, and Salop [1990], and Riordan [1998].

3 A notable exception is Hart and Tirole [1990], which considers the existence of asymmetric information before merger decisions take place.
B2. After observing the offered terms of trade, B1 and B2 make purchase decisions and then compete in a final goods market that exhibits downward sloping demand. S1 and S2 typically have the same commonly known production cost, the intermediate good either is homogeneous or is horizontally differentiated in a symmetric fashion such that neither product has a competitive advantage, and the downstream market exhibits similar features.

In this paper I depart from the standard approach in several ways by considering vertical mergers between a buyer and a seller in a duopoly procurement setting that features both incomplete information about costs, and asymmetries in sellers’ productive capabilities. Specifically, I consider the existence of only a single buyer who has unit demand, and I assume that the sellers’ production costs are privately known random draws from potentially different probability distributions. First, considering a single downstream buyer with unit demand abstracts both from “foreclosure” effects in the downstream market and from welfare effects through the elimination of double marginalization. This is similar to the approach in Bolton and Whinston [1993] that focuses on how effects in the intermediate goods market influence investment incentives. However, to the extent that the equilibrium outcomes change smoothly with the addition of downward sloping demand, merger-induced price effects would influence the prices that the downstream firm charges in the final goods market.

Second, allowing for incomplete information in this setting introduces strategic effects through merger-induced changes in the sellers’ price-setting behavior. With complete information about the sellers’ costs in this setting, a vertical merger would have no effect on prices, profits, or welfare, and there would be no incentive to merge. This qualitative difference is similar to that described in White [2007], which considers private information held by the buyer rather than the sellers, in a setting that otherwise is similar to mine. She concludes that the emergence of such differences indicates that allowing for incomplete information is an important consideration, rather than being simply a robustness check.

Third, allowing for asymmetries in the sellers’ likelihoods of having any particular cost permits the evaluation of issues such as determining the buyer’s most profitable merger partner, and assessing the effects on profits and welfare of the integrated and unintegrated sellers’ cost characteristics or relative sizes. It also is more realistic than is assuming symmetry, and it highlights certain conclusions that rely on symmetry assumptions. This is similar to the analysis in Linnemer [2003] that considers vertical integration involving a downstream industry populated by firms that differ in their commonly known production costs.

Two asymmetries in the model create technical difficulties that prevent an analytical comparison of premerger and postmerger equilibria. The first asymmetry arises from allowing differences to exist between the probability distributions that generate the sellers’ production costs, which would be empirically manifested as differences in premerger market shares. This asymmetry leads the sellers to set their premerger prices differently, for a given cost realization. The second asymmetry arises from the integrated seller’s setting its price at marginal cost, the efficient internal transfer price. This asymmetry leads the integrated and unintegrated sellers to set their postmerger prices differently, for a given cost realization, even if the probability distributions generating their production costs are the same.

I circumvent the analytic difficulties created by these asymmetries by using numerical techniques to determine the sellers’ premerger and postmerger equilibrium behavior. Next I apply the techniques by systematically computing the equilibria for a comprehensive set of over 10,000 configurations of the sellers’ cost distributions. Analyzing these data provides an understanding of the effects of vertical mergers that would not emerge from simply computing a small number of examples.4

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4The computational results focus on a duopoly setting, as otherwise the comparison of premerger and postmerger outcomes
The computational results yield several findings. First, many of the possible vertical mergers are jointly unprofitable, in the sense that the merged firm’s expected profits are less than the sum of the merging parties’ premerger expected profits. This unexpected finding stems from the pricing response of the unintegrated seller, who may profitably elect to increase price when confronted by price reductions from the integrated seller. Second, the buyer’s preferred merger partner almost always is the seller with lower expected costs, but this may be the smaller seller rather than the larger one. Interestingly, the buyer’s most profitable merger may not be the one that leads to the largest increase in joint profits, due to the concessions the sellers are willing to make to avoid being the sole unintegrated seller. Third, the unintegrated seller’s expected profits always decrease after the buyer’s preferred merger, sometimes dramatically. This finding suggests that increased exit incentives and diminished entry incentives are realistic consequences of vertical mergers in this environment. Finally, expected total welfare frequently decreases following the buyer’s preferred merger, although welfare is more likely to increase the larger is the buyer’s merger partner. The latter effect occurs because the vertical merger shifts market shares in a manner that reduces productive inefficiency that would occur absent the merger.

Some of my results contrast with unambiguous findings from two recent papers that examine similar duopoly settings. In a symmetric first-price auction setting, Choi [2007] examines rights of first refusal, while Burguet and Perry [2007] examine both rights of first refusal and vertical mergers. A right of first refusal has similar strategic effects as does a vertical merger, because in both instances the right-holding (or integrated) seller is able to effectively undercut its rival’s offer if it is profitable to do so. Both papers show that the contracting parties’ joint profits always increase and that total welfare always falls, while the issue of partner choice does not arise because of the symmetry assumption.

While it is not unusual for qualitatively different results to emerge from different treatments of vertical mergers, the driving force typically is a difference in the nature of the strategic interaction among market participants, such as the timing of moves or the available strategic variables. Here the underlying game is identical to the one in Choi [2007] and Burguet and Perry [2007], and the only difference is that I use less restrictive assumptions regarding the sellers’ cost distributions. The difference in results illustrates those assumptions’ otherwise unapparent qualitative importance.5

The rest of the paper is organized as follows. Section 2 introduces a standard duopoly price-setting game in which both sellers’ production costs are privately known. Section 3 considers a merger between a seller and a buyer that leads the integrated seller to set its price at marginal cost. Using computational methods, Section 4 describes the profitability and welfare effects of vertical mergers. Section 5 briefly concludes, and an Appendix describes the numerical techniques used to determine the premerger and postmerger equilibria.

2 Premerger Price-Setting

Consider a situation in which a prospective buyer of a product solicits price offers simultaneously from each of two sellers who produce homogeneous goods. Prior to making offers to the buyer, each seller $i$ privately draws its production cost, $c_i$, independently from the commonly known and differentiable cumulative distribution $F_i(c)$ with support $[c_\ell, c_\bar{c}]$. Assume that $F_i$ has a density $F_i'$ that is strictly positive on $[c_\ell, c_\bar{c}]$. The buyer would depend on how concentrated are the integrated seller’s horizontal rivals. This additional dimension would potentially cloud the main point of interest. However, the numerical techniques can be adapted for use in an oligopolistic postmerger setting.

5 In fact, earlier versions of the paper by Burguet and Perry [2007] considered a specific form of asymmetries that imposed stochastic dominance relationships between the distributions of the sellers’ costs. The more general approach to asymmetries that I employ leads to different qualitative conclusions from what they found in that earlier specification.
purchases from the low-priced seller at the offered price. In auction terminology, this is an asymmetric independent private value (IPV) first-price auction. Assume further that the buyer and sellers are all risk neutral, the number of firms is exogenous, and it is costless for sellers both to learn their production cost and to participate in the procurement process. The buyer’s next best supply alternative costs $c_B \geq c$, and the buyer’s profit from purchasing from one of the two sellers at price $p$ is $c_B - p$. Seller $i$’s profit from winning with price $p_i$ is $p_i - c_i$. I assume that the buyer can commit to the procurement format, which is a standard assumption in the auction literature.

The strategies employed in a Bayesian equilibrium of this game consist of mappings from sellers’ cost realizations to the prices that they offer. For any strictly increasing price-setting function $p_j(c)$ used by seller $j$, seller $i$ selects its price $p$ to maximize

$$
\pi_i(p|c) = (p - c) \left(1 - F_j(\varphi_j(p))\right),
$$

where $\varphi_j(p)$ is the inverse of $p_j(c)$. The first term of $\pi_i(p|c)$ is the margin seller $i$ receives if it wins the contract, while the second term is the probability that seller $i$ wins the contract. Seller $i$ wins if and only if seller $j$’s cost is such that its price exceeds $p$. The respective first-order conditions for sellers 1 and 2 are

$$1 - F_2(\varphi_2(p)) - (p - c) F_2'(\varphi_2(p)) \varphi_2^2(p) = 0 \quad (1)$$

and

$$1 - F_1(\varphi_1(p)) - (p - c) F_1'(\varphi_1(p)) \varphi_1(p) = 0. \quad (2)$$

In equilibrium, seller $i$’s cost must be $\varphi_i(p)$ when it sets price $p$, so the preceding first-order conditions can be rearranged to yield the set of simultaneous ordinary differential equations

$$
\begin{align*}
\varphi_2'(p) &= \frac{1 - F_2(\varphi_2(p))}{(p - \varphi_2(p)) F_2'(\varphi_2(p))} \quad (3) \\
\varphi_1'(p) &= \frac{1 - F_1(\varphi_1(p))}{(p - \varphi_2(p)) F_1'(\varphi_1(p))}
\end{align*}
$$

with appropriate boundary conditions.

An analytic solution to the system (3) exists only for the symmetric case in which both sellers draw their costs from the same distribution $F$. In the asymmetric case, the lack of analytic solutions prevents one from finding analytic solutions for the expected price and the sellers’ expected profits.

A number of authors have theoretically examined equilibrium behavior in the asymmetric case. Notably, in the setting I use here Lebrun [1999] proves that an equilibrium exists in which the sellers employ strictly increasing and differentiable price-setting functions. Bajari [2001] proves that the equilibrium is unique. Without imposing further assumptions, the consensus in the literature appears to be that little more can be done to make quantitative or qualitative claims about the sellers’ behavior.

To make headway, theoretical characterizations of equilibrium behavior assume that the sellers’ cost distributions are related through first-order stochastic dominance. Once this assumption is made, a number

\[ \text{The assumption of inelastic demand holds in many settings, particularly in intermediate goods markets in which the component being sought is a small part of the cost of the final good. The inelastic demand assumption also is typically used in modeling government procurement. However, there exist other settings in which firms set their prices using first-price rules, but in which demand for the product has some elasticity. Spulber [1995] analyzes this setting assuming the sellers’ costs all are from the same distribution. The numerical techniques used in Section 4 should be adaptable for use in the asymmetric setting.} \]

\[ \text{\[ F_i(c) \text{ first-order stochastically dominates } F_j(c) \text{ if } F_i(c) \leq F_j(c) \text{ for all } c. \] One consequence of this relationship is that the expected cost under distribution } F_i(c) \text{ is greater than the expected cost under distribution } F_j(c). \] \]
of interesting qualitative equilibrium properties emerge. For example, Waehrer [1999] shows that a seller with lower average costs sets its price less aggressively than a seller with higher average costs, in the sense that its price-setting function specifies a higher price for a given cost. Lebrun [1998] shows that if one seller’s cost distribution shifts to make it more likely that it has low costs, then each seller’s price distribution shifts to put more weight on lower prices.

As an alternative to the theoretical approach, other authors have used numerical techniques to evaluate various issues in asymmetric first-price auction markets. Marshall, Meurer, Richard, and Stromquist [1994] provide the first numerical analysis of asymmetric first-price auctions in order to evaluate the effect of coalitions. Dalkir, Logan, and Masson [2000] numerically analyze asymmetric first-price auctions in order to evaluate the effect of horizontal mergers. They provide examples to show that inappropriately using symmetric models may severely overstate an acquisition’s price effect.

One potential drawback of most of the published numerical analyses of asymmetric first-price auctions is that attention is restricted to settings in which sellers draw their costs from power families, in which each seller’s cost is the minimum of a firm-specific number of draws from a cost distribution common to all sellers. In such settings, seller i’s cost distribution is \( F_i(c) = 1 - (1 - F(c))^{k_i} \), where \( k_i \) is a positive real number that indicates i’s number of draws from the common distribution \( F \). One interpretation provided to justify the use of power families is that \( k_i \) can be equated with some measure of seller i’s size, capacity, or expertise. While this interpretation may be appropriate in some instances, one consequence of using power families is that the sellers’ cost distributions are necessarily related through first-order stochastic dominance.

Therefore, any analysis using power families is limited in a potentially important way. The numerical approach I use in Section 4 dispenses with power families and the first-order stochastic dominance assumption used in prior studies. As will be shown, this generalization can lead to important qualitative changes in the results.

### 3 Postmerger Price-Setting and the Choice of Merger Partner

Consider the effect of a merger between seller 1 (the integrated seller) and the buyer on the competition for the buyer’s business. The integrated seller sets its price at marginal cost, the efficient internal transfer price. Consequently, if the unintegrated seller offers a price \( p \), then it wins the contract if and only if \( c_I \geq p \), where the subscript “I” denotes the integrated seller. In this postmerger setting, the unintegrated seller chooses its price \( p \) to maximize

\[
\pi_U(p|c) = (p - c)\left[1 - F_I(p)\right],
\]

where the subscript “U” denotes the unintegrated seller. The first term of \( \pi_U(p|c) \) is the unintegrated seller’s margin if it wins the contract, while the second term is the probability that it wins the contract. The first-order condition for the unintegrated seller is

\[
1 - F_I(p) - (p - c)F'_I(p) = 0,
\]

which determines the unintegrated seller’s optimal price offer as a function of its cost and the integrated seller’s cost distribution.

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8To be more precise, the author assumes the distributions satisfy “Multivariate Positivity of Order 2,” which is equivalent to the monotone likelihood ratio property, and which implies first-order stochastic dominance.

9For example, see Waehrer and Perry [2003].
Recalling equation (2), which gave the first-order condition for the unintegrated seller’s premerger profit maximization problem (with the subscript “I” substituted for “1” as appropriate),

\[ 1 - F_I (\varphi_I(p)) - (p - c)F'_I (\varphi_I(p)) \varphi'_I(p) = 0, \] (2)

it is clear that the unintegrated seller’s first-order conditions for its optimal price differ premerger and postmerger, unless \( \varphi_I(p) \equiv p \). However, it is not evident from equations (2) and (4) whether the unintegrated seller sets its price higher or lower postmerger than it did premerger. For this reason, the direction and magnitude of the effect on the integrated seller’s expected profit cannot be determined analytically. Moreover, while one can show that the unintegrated seller’s expected profit falls postmerger,\(^{10}\) this effect’s magnitude cannot be derived analytically. Consequently, the economic significance of the profit loss borne by the unintegrated seller is unclear.

While the preceding analysis holds for each possible merger, in practice the buyer will consummate the merger that it determines to be most profitable. In fact, it may choose not to merge at all if the available mergers are jointly unprofitable. Rather than assessing the buyer’s choice of a merger partner by analyzing a formally specified bargaining game among the buyer and the two sellers, I simply argue that the buyer’s most profitable merger partner is the seller with the higher willingness to pay to purchase the buyer. Seller \( i \)’s willingness to pay is the difference between the integrated firm’s expected profit if seller \( i \) is the merger partner \( (\pi^I_i) \) and seller \( i \)’s expected profit if it is the unintegrated seller \( (\pi^U_i) \).\(^{11}\) Suppose that seller 1’s willingness to pay is larger than seller 2’s, but that the buyer is considering merging with seller 2 according to an agreement that provides seller 2 with a payoff of \( V_2 \geq \pi^U_2 \) and the buyer with a payoff of \( \pi^I_2 - V_2 \).\(^{12}\) Seller 1 can profitably offer the buyer a payoff of \( \pi^I_2 - V_2 + \epsilon \) for some \( \epsilon > 0 \). The buyer obviously would do better by accepting seller 1’s offer. To see why seller 1 would do better to make such an offer, consider the circumstances under which its payoff from making the offer exceeds its payoff from not making the offer,

\[ \pi^I_1 - (\pi^I_2 - V_2 + \epsilon) > \pi^U_1 \iff \pi^I_1 - \pi^U_1 > \pi^I_2 - V_2 + \epsilon. \]

The second inequality holds, for sufficiently small \( \epsilon \), because seller 1’s willingness to pay exceeds seller 2’s. Thus, for any agreement that the buyer and seller 2 might consider, seller 1 can profitably make a better offer to the buyer when seller 1’s willingness to pay exceeds seller 2’s.

The preceding argument specifies which seller is the buyer’s preferred merger partner, but does not speak to whether or not a vertical merger actually would occur. For example, the maximum of the sellers’ willingnesses to pay must exceed the buyer’s premerger expected profit, or the buyer would not wish to merge. While a complete characterization of the conditions under which a vertical merger will occur may be difficult to specify, a sufficient condition is that at least one of the two available mergers is jointly profitable. If so, then the buyer and that seller can split the profits from merging in a way that makes each better off than without the merger. In the computational results that follow, at least one of the two available vertical mergers always is jointly profitable.

On a final note, the effect of incomplete information on the preceding results bears mentioning. If the

\(^{10}\)This result follows from a standard revealed preference argument and the fact that the integrated seller’s postmerger price-setting behavior is “more aggressive,” in the sense that any price the unintegrated seller sets postmerger is more likely to lose the competition than it was premerger.

\(^{11}\)One can show that a seller’s willingness to pay is always positive, because a seller’s premerger profits always exceed its profits as the unintegrated seller, and are always less than its profits as the integrated seller.

\(^{12}\)The constraint on \( V_2 \) reflects that seller 2 is not willing to receive a payoff lower than what it would earn by not being the merger partner.
sellers’ costs were commonly known before they submitted price offers to the buyer, then a vertical merger would have no effect on expected profits or expected total welfare. The reason is that the winning seller will be the one with the lower cost, both premerger and postmerger. When the unintegrated seller wins, it will pay a price just below its rival’s cost. Thus, its expected profits are unaffected by the merger. Similarly, the merged firm’s joint profits will be unaffected. Consequently, expected total welfare also will be unaffected. The results change in the presence of incomplete information because the merger leads to changes in both how the sellers set their prices, and their likelihood of winning the procurement contest.¹³

4 The Profitability and Welfare Effects of Vertical Mergers

Because the theoretical results in Sections 2 and 3 are analytically intractable, they provide little insight into the qualitative and quantitative effects of vertical mergers in the procurement setting modeled here. However, such insights clearly are critical in understanding how the strategic changes caused by a vertical merger influence competition. This section complements the theoretical analysis in Sections 2 and 3 by evaluating merger effects using a computational approach.¹⁴ Specifically, I systematically compute the premerger and postmerger equilibria for an entire family of cost distributions, an approach that provides comprehensive premerger and postmerger data on expected profits and expected total welfare.

I compute the premerger and postmerger equilibria for each \((\mu_1, \sigma^2_1, \mu_2, \sigma^2_2)\)-tuple in a grid of the four-dimensional parameter space. The cost distributions all are from the family of “truncated and modified” Beta distributions that are described in the Appendix.¹⁵ Figure 1 shows in dark circles the set of feasible means and variances, on the grid, of the truncated and modified Beta distribution. The clear circles represent additional means and variances on the grid that are feasible for the standard Beta distribution but that are not feasible for the truncated and modified Beta distribution. As is evident, the truncated and modified Beta distribution is very close to the standard Beta distribution, in terms of the feasible set of means and variances.

Subject to the parameter restrictions and the grid size, I consider 101 distributions.¹⁶ Thus, I compute premerger and postmerger equilibria for 10,201 \((101 \times 101)\) combinations of the sellers’ cost distributions. Separate programs, whose essential features are described in the Appendix, computed the premerger and postmerger equilibria, and they also computed the expected price as well as each seller’s expected profit and probability of winning. The latter can be equated with each seller’s market share.

Subsection 4.1 analyzes premerger and postmerger profits, and the profitability off all vertical mergers. It may be skipped by readers wishing to move immediately to subsection 4.2’s analysis of the buyer’s preferred

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¹³This contrasts with the finding in Bikhchandani, et al [2005], who evaluate rights of first refusal in a symmetric environment using second-price auction rules. In an IPV setting, one typically considers a second-price auction with incomplete information to have the same outcome as a first-price auction with complete information. The difference in predictions emerges because in that paper, postmerger, the right-holding seller is allowed to meet the price that the putative winner amongst its rivals would pay, which is actually the second-lowest cost of its rivals. This auction format severely disadvantages the rival that otherwise would have won, because it is never allowed to submit a price offer that it is willing to pay.

¹⁴Researchers have previously turned to computation to obtain both qualitative and quantitative insights from models that are analytically cumbersome or intractable. For example, Quirmbach [1993] provides computational evidence on the relationship between the incentives to perform R&D and the extent of post-innovation collusive behavior. Haubrich [1994] provides computational evidence that helps bridge the gap between the theory and the reality of executive compensation.

¹⁵For now it is sufficient to know that, in order to satisfy the equilibrium existence conditions from Lebrun [1999], the support of the standard Beta distribution is truncated slightly and that a small amount is added to the density.

¹⁶The value of the mean runs from 0.05 to 0.95, with a grid size of 0.075. The value of the variance runs from 0.02 to 0.23, with a grid size of 0.0175.
vertical merger.

4.1 Analysis of All Vertical Mergers

As a first step I investigate the effects of all vertical mergers, deferring until the next subsection the buyer’s choice of a merger partner. Specifically, I evaluate a vertical merger between the buyer and seller 1 for each combination of feasible means and variances for both sellers. This approach covers all possible mergers, because by construction every configuration of parameters has a corresponding configuration in which the specification of the sellers’ cost distributions is reversed.

Before assessing the changes in profits caused by a vertical merger, in Results 1 and 2 I first characterize premerger and postmerger expected profits. While readers who wish to move directly to the assessment of a vertical merger’s profitability can skip directly to Result 3, I provide these intermediate results for two reasons. First, premerger and postmerger expected profits are the constituent elements that influence merger profitability. Second, the characterization of premerger and postmerger expected profits is of independent interest, because the computational results provide novel evidence about how expected profits are influenced by the distributions of the sellers’ costs. These results potentially could be used to provide guidance about fruitful avenues for future theoretical research on asymmetric auctions, or as inputs into analyses of strategic decisions such as technology choice, market entry, or vertical divestiture.

While some of the results to follow come from examining the raw data from each combination of parameters, I also evaluate the data by approximating the nonlinear functions that determine expected profits. I use the least squares criterion to fit the computed data to a second-order polynomial in the mean and the variance of each seller’s cost distribution. For example, the equation for the buyer’s premerger expected profit is

$$\pi_B = \alpha_0 + \alpha_1 \mu_1 + \alpha_2 \sigma_1^2 + \alpha_3 \mu_2 + \alpha_4 \sigma_2^2 + \alpha_5 \mu_1^2 + \alpha_6 \mu_1 \sigma_1^2 + \alpha_7 \mu_1 \mu_2 + \alpha_8 \mu_1 \sigma_2^2 + \alpha_9 (\sigma_1^2)^2 + \alpha_{10} \sigma_1^2 \mu_2 + \alpha_{11} \sigma_1^2 \sigma_2^2 + \alpha_{12} \mu_2^2 + \alpha_{13} \mu_2 \sigma_2^2 + \alpha_{14} (\sigma_2^2)^2.$$  

The same equation also is fitted to other profit measures.

The curve-fitting technique usefully summarizes how expected profits are influenced by the means and variances of the sellers’ cost distributions, to the extent that the curve fits well. Moreover, from the polynomial approximations one can easily perform comparative statics involving the primitives of the sellers’ cost distributions. The effect of increasing any one of the parameters is found by differentiating the fitted equations with respect to that parameter, and then evaluating the expressions for particular values of the sellers’ means and variances that are elements of the feasible set of means and variances for the truncated and modified Beta distribution. General patterns in the signs of these derivatives, evaluated at all points in the grid of the parameter space, provide evidence about the influence of changes in the cost distributions’ primitives.

An explanation of the language used to describe the comparative static results is in order. I say that an effect occurs “always” if it occurs in 100 percent of the cases, “almost always” if it occurs in more than 95 percent of the cases, and “typically” if it occurs in between 50 percent and 95 percent of the cases.

Result 1 Consider the premerger expected profits of seller 1 and the buyer, \(\pi_1\) and \(\pi_B\).

- Seller 1’s expected profits almost always decrease with increases in the mean of its cost, always increase with increases in the variance of its cost, and typically increase with increases in the mean or variance
of seller 2’s cost.

- The buyer’s expected profits almost always decrease with increases in the mean of a seller’s cost, and typically decrease with increases in the variance of a seller’s cost.

- The sum of seller 1’s and the buyer’s expected profits always decreases with increases in the mean of seller 1’s cost, typically increases with increases in the variance of seller 1’s cost, always decreases with increases in the mean of seller 2’s cost, and almost always decreases with increases in the variance of seller 2’s cost.

**Evidence:** The first three columns of Table 1 report the curve-fitting results for seller 1’s and the buyer’s premerger expected profits, and their sum. Using the coefficients from the curve-fitting results, at each point in the grid of the parameter space one can evaluate the relevant derivatives of the fitted quadratic equation. The first three columns of Table 2 report the fraction of such derivatives that are strictly positive. The derivatives of $\pi_1$ with respect to $\mu_1, \sigma^2_1, \mu_2,$ and $\sigma^2_2$ are strictly positive for 3.73%, 100.00%, 93.96%, and 81.03% of the 10,201 configurations considered, while the derivatives of $\pi_B$ are strictly positive for 1.27%, 6.27%, 1.27%, and 6.18% of the configurations. The slight differences in the effects on $\pi_B$ of increasing $\sigma^2_1$ versus $\sigma^2_2$ reflect that a finite number of random draws were employed to calculate expected profits. Finally, the derivatives of $\pi_1 + \pi_B$ are strictly positive for 0.00%, 76.91%, 0.00%, and 0.57% of the configurations.

While the main reason for presenting Result 1 is to contribute to the subsequent discussion of merger profitability, it is worth digressing briefly to relate some of its findings to existing analytic results involving changes in sellers’ cost distributions. Lebrun [1998] shows that if one seller’s cost distribution improves, in the sense of first-order stochastic dominance, then its rival’s expected profit decreases and the buyer’s expected profit increases. Interestingly, the improved seller’s expected profit actually falls for any particular cost realization, but there is no unambiguous result regarding its ex ante expected profit. In a model in which sellers’ costs are distributed discretely rather than continuously, Thomas [1997] shows that the improved seller actually can be worse off due to more aggressive price-setting by its rivals.

Thinking of an “improvement” as a reduction in the mean of a seller’s cost, Result 1 shows that the rival is not always made worse off in this setting, nor is the buyer always made better off. Moreover, the improved seller is made better off with much greater frequency than one might expect given the results in Thomas [1997]. One distinction to consider between the earlier papers and the present one is that the earlier papers considered distributional changes that, in at least some instances, involved changing simultaneously the mean and the variance of a seller’s cost. Here I can evaluate the effects of changing just one of those parameters.

The sum of the premerger profits of seller 1 and the buyer provide the relevant comparison to the integrated firm’s postmerger expected profit. The negative effect of increasing the mean of the integrated seller’s cost is unsurprising given its negative effects on the seller’s and the buyer’s individual premerger expected profits. The positive effect on the seller’s expected profit from increasing the variance of its cost generally outweighs the negative effect on the buyer’s expected profit. However, the positive effects on the seller’s expected profits from increasing either the mean or the variance of its rival’s cost generally are outweighed by the negative effects on the buyer’s expected profit.

**Result 2** Consider the postmerger expected profits of seller 2 following a merger between seller 1 and the buyer, $\pi^U_2$, and of the integrated firm following a merger between seller 1 and the buyer, $\pi^I_1$. 

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The unintegrated seller’s expected profits typically decrease with increases in the mean of its cost, always increase with increases in the variance of its cost, and typically increase with increases in the mean or variance of the integrated seller’s cost.

The integrated firm’s expected profits always decrease with increases in the mean of the integrated seller’s cost, almost always decrease with increases in the variance of the integrated seller’s cost, almost always decrease with increases in the mean of the unintegrated seller’s cost, and always increase with increases in the variance of the unintegrated seller’s cost.

Evidence: The fourth and fifth columns of Table 1 report the curve-fitting results for the integrated firm’s expected profit and the unintegrated seller’s expected profit, with sellers 1 and 2 respectively in the roles of the integrated and unintegrated sellers. As with the similar findings from Result 1, the relevant derivatives from the fitted quadratic equation are evaluated using the coefficients from the curve-fitting results. The fourth and fifth columns of Table 2 report the fraction of such derivatives that are strictly positive. The derivatives of $\pi_U^2$ are strictly positive for 5.30%, 100.00%, 90.79%, and 93.70% of the configurations considered, while the derivatives of $\pi_I^1$ with respect to $\mu_1, \sigma^2_1, \mu_2,$ and $\sigma^2_2$ are strictly positive for 0.00%, 3.46%, 1.66%, and 100.00% of the 10,201 configurations.

For the unintegrated seller there is little qualitative difference in how its premerger and postmerger expected profits are affected by changes in the sellers’ cost distributions. In both cases, increasing the mean of its cost generally is harmful, increasing the variance of its cost generally is beneficial, and increasing the mean or variance of its rival’s cost generally is beneficial. The one potentially confusing finding is that the unintegrated seller’s profits do not always decrease as its mean cost increases. However, recall that the unintegrated seller charges the same price postmerger for a given cost realization, regardless of its mean cost. Consequently, any change in the unintegrated seller’s expected profit is driven solely by changes in the density of its cost distribution, and there is no general result that increasing the mean necessarily leads to a change in the density that is unfavorable from the perspective of expected profits.

For the integrated profits of seller 1 and the buyer, there is no substantial change in the effect of changing the mean cost of seller 1 or seller 2. However, there is a substantial change in the effect of changing the variance of each seller’s cost. Increasing the variance of the integrated seller’s cost lowers joint profits postmerger, but premerger it tends to increase them. In contrast, increasing the variance of the unintegrated seller’s cost always increases joint profits postmerger, but premerger it almost always decreases them.

Having provided information about premerger and postmerger profit levels, I now use that information to examine the merger’s effect on the merging parties’ joint expected profits.

Result 3 Consider the profitability of a vertical merger between seller 1 and the buyer, $\pi_I^1 - (\pi_1 + \pi_B)$.

- The merging parties’ joint expected profits increase in approximately 81 percent of the cases examined. The average percentage change is an 8 percent increase, ranging from a 22 percent decrease to a 52 percent increase.

- Merger profitability almost always decreases with increases in the mean or variance of the integrated seller’s costs, and almost always increases with increases in the mean or variance of the unintegrated seller’s costs.

- The sellers’ premerger market shares poorly predict the magnitude of merger profitability, but do provide information about whether the merger will be jointly profitable or unprofitable.

10
Evidence: Examination of the raw data reveals that the merged firm’s joint expected profits increase for 8,230 of the 10,201 configurations of the sellers’ cost distributions, and that the average percentage change is an 8 percent increase. The smallest change is a 21.97% reduction, and the largest is a 51.54% increase. The sixth column in Table 1 reports the curve-fitting results for the profitability of a vertical merger between seller 1 and the buyer. Using these curve-fitting results, the sixth column of Table 2 reports the fraction of feasible \((\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)\)-tuples for which the derivatives of the change in the merged firm’s joint expected profits are strictly positive. The derivatives of \(\pi'_1 - (\pi_1 + \pi_B)\) with respect to \(\mu_1, \sigma_1^2, \mu_2, \) and \(\sigma_2^2\) are strictly positive for 1.66%, 2.67%, 99.34%, and 99.78% of the 10,201 configurations considered. Figure 2 reports the change in joint profits as a function of the integrated seller’s premerger expected market share, which shows that there is no apparently strong monotone relationship between market share and merger profitability. To see if there exists a useful statistical relationship, I have regressed the change on a 2nd-order polynomial of the integrated seller’s premerger market share. The \(R^2\) of the regression is 0.3702, so market shares explain little of the change in the merging parties’ joint expected profits. However, if the merging seller’s premerger market share exceeds 50%, then note that the merger overwhelmingly is jointly profitable.

Figure 2 Here

The unexpected finding that a vertical merger can be jointly unprofitable stems from changes in the unintegrated seller’s price-setting. One can show that if the unintegrated seller sets weakly lower prices postmerger for all cost realizations, then the merged firm’s expected profits must weakly increase. Consequently, the observed reductions in joint profits imply that the unintegrated seller responds to the integrated seller’s lower prices by strictly increasing price postmerger, for at least some cost realizations. While initially it might seem unusual for the unintegrated seller to increase its price when confronted by the postmerger reduction in the integrated seller’s price, it occurs for precisely the same reason that a monopolist’s price might increase if demand decreases. Namely, the unintegrated seller’s optimal price depends not just on the probability of winning with a particular price, but also on how that probability changes as its price changes.

It is also worth considering the merger’s profit effects in light of the results from the symmetric models in both Choi [2007] and Burguet and Perry [2007] that the contracting parties’ joint expected profits always increase. The difference I find in the profitability effects can be understood by partitioning the problem into four cases, depending on whether the integrated seller would have won or lost, both premerger and postmerger. Consider an arbitrary pair of cost draws for the two sellers. If the integrated seller would have won both premerger and postmerger, then the merged firm’s joint profits are \(c_B - c_I\) both premerger and postmerger. This result is the same in both the symmetric and the asymmetric settings. If the integrated seller would have won premerger and lost postmerger, then the merged firm’s joint profits increase. Premerger joint profits are \(c_B - c_I\), while postmerger profits are \(c_B - p_U\), and we know that \(p_U < c_I\) in order for the unintegrated seller to win postmerger. This result also is the same in both the symmetric and asymmetric settings. Finally, if the integrated seller would have lost premerger, and either won or lost postmerger, then the effect on the merged firm’s joint profits generally is ambiguous. This ambiguity arises because there is no general relationship between the two sellers’ premerger price-setting functions. In the symmetric settings considered here, the net effect over both cases is positive, which is consistent with the findings from the symmetric models in Choi [2007] and Burguet and Perry [2007]. However, the present results show that conclusion does not hold more generally.

Now consider the comparative statics involving the primitives of the cost distributions. For an increase in the mean of the integrated seller’s cost, the negative effect on the integrated firm’s postmerger expected
profit generally outweighs the negative effect on premerger joint profits. For an increase in the variance of the integrated seller’s cost, the generally negative effect on merger profitability is unsurprising given the effects on premerger and postmerger expected profits. For an increase in the mean of the unintegrated seller’s cost, the negative effect on postmerger integrated expected profits generally is outweighed by the negative effect on premerger joint profits. For an increase in the variance of the unintegrated seller’s cost, the generally positive effect on merger profitability is unsurprising.

Figure 3 helps to visualize the comparative statics from the four-dimensional parameter space by illustrating merger profitability as a function of differences in the sellers’ means and differences in the sellers’ variances. The “+” marks represent \((\mu_1 - \mu_2, \sigma_1^2 - \sigma_2^2)\) pairs for which a merger involving seller 1 is jointly profitable, while the “×” marks represent \((\mu_1 - \mu_2, \sigma_1^2 - \sigma_2^2)\) pairs for which a merger involving seller 1 is jointly unprofitable. Note that there may exist several different parameter configurations that yield the same \((\mu_1 - \mu_2, \sigma_1^2 - \sigma_2^2)\) pair, and those different configurations may yield different answers regarding merger profitability. Thus, pairs marked only by a “+” are those for which the merger always is profitable, pairs marked only by a “×” are those for which the merger always is unprofitable, and pairs marked by both “+” and “×” have some mergers that are profitable, and some mergers that are unprofitable. The main point to take away from Figure 3 is that vertical mergers always are jointly unprofitable when the integrated seller has a high mean and high variance, relative to the unintegrated seller. Also, vertical mergers always are jointly profitable if the integrated seller’s mean cost is less than the unintegrated seller’s mean cost, regardless of the differences in the variances of their costs.

Finally, the market share results suggest that simply knowing a potential partner’s size is insufficient for gauging the magnitude of a vertical merger’s profitability. When combined with the curve-fitting results, this suggests some information about potential partners’ characteristics is needed for assessing the magnitude of profit gains or losses. However, note that mergers involving a seller with a market share greater than 50 percent almost always are jointly profitable.

4.2 Analysis of the Buyer’s Preferred Vertical Merger

Having provided some insights about the effects of vertical mergers in general, I now consider the buyer’s choice of a merger partner. As described in Section 3, the buyer will find it most profitable to merge with the seller with the higher willingness to pay, provided that willingness to pay exceeds the buyer’s premerger expected profit. Because there are so many jointly unprofitable mergers, however, it is not clear whether or not the buyer actually will wish to merge with one of the sellers. Analyzing merger decisions for all configurations of the sellers’ costs yields the following.

Result 4 Consider the buyer’s most profitable vertical merger.

- For every configuration of the sellers’ cost distributions, the buyer can profitably merge with one of the sellers.

- The buyer’s most profitable merger partner is the seller with lower expected costs in approximately 97 percent of the cases in which the sellers’ expected costs differ, and is the larger seller in approximately 87 percent of the cases in which the sellers are asymmetric.
• In approximately 27 percent of the asymmetric settings, the buyer’s most profitable merger is not the merger yielding the greatest increase in the merging parties’ joint profits.

Evidence: As was argued earlier, the seller with the higher willingness to pay is the buyer’s most profitable merger partner. Examination of the raw data reveals that the maximum of the two sellers’ willingnesses to pay always exceeds the buyer’s premerger expected profit, and that same merger always is jointly profitable. Hence, the buyer and its most profitable merger partner always can find a mutually agreeable split of the profits associated with the vertical merger. There are 9,208 configurations of the sellers’ costs for which the two sellers’ expected costs differ. Examination of the raw data reveals that, in 8,972 of those cases, the seller with the lower expected cost has a higher willingness to pay. Similarly, the most profitable merger partner’s market share weakly exceeds 50 percent in 8,797 of all 10,100 cases in which the sellers are asymmetric. Finally, in 7,393 of the 10,100 asymmetric settings, the merger with the seller with the greater willingness to pay corresponds to the merger yielding the largest increase in the parties’ joint expected profits. ■

Although Result 3 revealed that almost twenty percent of vertical mergers are jointly unprofitable, it turns out that at least one of the two mergers available is jointly profitable. Hence, in all cases considered a merger will occur. As shown in Figure 2, a merger almost always is profitable if the integrated seller’s market share exceeds 50 percent. When the two sellers have different premerger shares, obviously one of them will have a market share in excess of 50 percent.

A seller’s mean cost advantage and its premerger share both seem to be appropriate measures of its relative competitive position. However, the second part of Result 4 illustrates that while both are good predictors of willingness to pay, the mean cost advantage is more reliable in the setting considered here. Of course, the sellers’ relative sizes may be more easily measured.

To help illustrate the comparative statics of the sellers’ relative willingnesses to pay to be the merger partner, Figure 4 shows the preferred merger partner as a function of the difference in both the means and the variances of the sellers’ costs. The only instances in which a seller can be the preferred partner while having a higher mean cost than its rival is when its mean cost is not much higher than its rival’s, and its variance either is higher or is much lower.

Figure 4 Here

Interestingly, the vertical merger that occurs is not necessarily the one yielding the larger increase in the merging parties’ joint expected profits. This apparent puzzle can be resolved by recognizing that determining the most profitable merger partner utilizes postmerger profits, while determining merger profitability utilizes postmerger and premerger profits. For example, suppose that seller 1 is the buyer’s preferred merger partner, while seller 2 is the merger partner leading to the larger increase in the parties’ joint expected profits. The first condition can be written as

$$\pi_1^I - \pi_1^U > \pi_2^I - \pi_2^U,$$

while the second can be written as

$$\pi_2^I - (\pi_2 + \pi_B) > \pi_1^I - (\pi_1 + \pi_B).$$

These two conditions respectively can be rewritten as

$$\pi_1^I - \pi_2^I > \pi_1^U - \pi_2^U.$$
and
\[ \pi_1 - \pi_2 > \pi_1^I - \pi_2^I. \]
The first condition says that seller 1 is the preferred merger partner if seller 1 is relatively more profitable than seller 2 as the integrated rather than as the unintegrated seller. The second condition says that merging with seller 2 yields a greater increase in joint profits if seller 2 is relatively more profitable than seller 1 as the integrated seller rather than in the premerger setting. Both conditions can be satisfied provided that
\[ \pi_1 - \pi_2 > \pi_1^I - \pi_2^I > \pi_1^U - \pi_2^U, \]
and nothing precludes the existence of such a relationship.

Finally, I now discuss the most profitable vertical merger’s effect on both the unintegrated seller and expected total welfare. The selection effect created by the buyer’s choice makes less informative the sort of curve-fitting and comparative statics results presented in subsection 4.1’s analysis of all vertical mergers, so instead I relate the effects to the sellers’ relative sizes by using the integrated seller’s premerger market share.

**Result 5** The buyer’s most profitable merger always decreases the unintegrated seller’s expected profit. The average reduction is 58 percent, with a range from 3 percent to 100 percent. The percentage loss in expected profits tends to be greater the larger is the merger partner’s premerger market share.

**Evidence:** Examination of the raw data reveals that the unintegrated seller’s expected profit always is lower postmerger than it was premerger, and that the average change is a 58% reduction, ranging from a 3% decrease to a 100% decrease. The loss in expected profits tends to be greater with increases in the integrated seller’s premerger market share. ■

Result 5 reveals that the unintegrated seller experiences a substantial profit reduction following the buyer’s preferred merger. This suggests that concerns about the exit-inducing or entry-deterring effects of vertical mergers are well-founded in this setting. While one might argue that it is not in the buyer’s interest to remove a potential supplier, that consideration may be dominated by the increased profits of the integrated supplier in competition for other buyers’ business. It seems that such a concern on the part of antitrust authorities would be more pronounced the more important is the merging buyer’s business to the unintegrated seller, relative to the business available from other buyers.

**Result 6** The buyer’s most profitable merger always leads to the highest level of postmerger expected total welfare. However, the buyer’s most profitable merger decreases expected total welfare from its premerger level in approximately 63 percent of the cases examined. Postmerger expected welfare tends to be higher the larger is the merger partner’s premerger market share.

**Evidence:** Suppose that the buyer’s most profitable merger is with seller 1. As was argued earlier, this is the case when seller 1’s willingness to pay exceeds seller 2’s. Thus, we have
\[ \pi_1^I - \pi_1^U > \pi_2^I - \pi_2^U. \]
The preceding expression is equivalent to
\[ \pi_2^I + \pi_2^U > \pi_2^I + \pi_1^U, \]
where the left hand side of the expression is expected total welfare when seller 1 is the merger partner, and the right hand side is expected total welfare when seller 2 is the merger partner. Therefore, the buyer’s most profitable merger leads to the highest postmerger expected total welfare. However, examination of the raw data reveals that expected total welfare decreases from its premerger level in 6,387 of the 10,201 case considered. Figure 5 reports the change as a function of the integrated seller’s premerger market share, which reveals that vertical mergers tend to increase welfare when the integrated seller is larger.

One factor that likely influences Result 6’s seemingly high frequency and magnitude of welfare losses is that eliminating double-marginalization has no impact in the present setting, in contrast to the case in settings with more elastic demand. Because demand is perfectly inelastic, the negative welfare effects instead arise from merger-induced reductions in productive efficiency. These reductions can be understood by considering the inefficiencies associated with price-setting in asymmetric procurement environments. Specifically, inefficiencies occur because sellers with low expected costs tend to set prices less aggressively than do sellers with high expected costs, in order to exploit the relatively weak competition they face. Hence, sellers with low expected costs tend to win less frequently than they should premerger, from an efficiency perspective. One can show from the data that the integrated seller’s market share increases postmerger. Thus, if the merger partner is small, the inefficiency is exacerbated because its wins postmerger even more frequently than it did premerger. The reverse holds if the merger partner is large. That is, it wins more frequently than it did premerger, which tends to reduce the extent of inefficiency. In terms of predicting harmful merger effects, it therefore appears that mergers are less likely to cause welfare losses the larger is the integrated seller. This finding is similar to that found with the asymmetric downstream Cournot setting in Linnemer [2003], and it contrasts with the recommendation from Hart and Tirole [1990] that a vertical merger merits particularly close scrutiny when the merger partner is large.

Figure 5 Here

5 Conclusion

In this paper I quantify the profitability and welfare effects of vertical mergers in a duopolistic procurement setting in which the sellers’ costs are independently and privately drawn from different continuous distributions. The computational results yield insights for the buyer, the sellers, and regulatory officials. First, many mergers are jointly unprofitable, but the buyer always can profitably merge with one of the sellers. When selecting its merger partner, the buyer may find the smaller seller to be a more profitable choice. Interestingly, the buyer’s preferred merger may not be the one yielding the greatest increase in the parties’ joint profits. Second, concerns about the exit of rivals and about welfare losses in this setting appear to be well-founded. A seller experiences a dramatic profit reduction if its rival integrates, and welfare falls for more than half of the cases considered. In terms of assessing likely harm, expected total welfare tends to increase postmerger if the buyer’s merger partner is sufficiently large premerger.

The framework developed here to evaluate vertical mergers also can be used to evaluate other vertical relationships, such as contracts providing a seller with a right of first refusal, and bribery of an auctioneer by one of the bidders. In addition, the curve-fitting results that provide reduced-form profit functions could be used to consider the effects of merger-induced (in)eiciencies, or as inputs into analyses of other strategic issues in procurement. Finally, the present framework could be extended to consider issues such as the effect of multiple buyers on the ultimate extent of vertical integration. While the merger-related effects for any
single buyer will follow from the present analysis, if buyers differ then sellers might have varying preferences about with which buyer they merge. Also, the merger negotiations would become more complicated, because the sellers’ outside options would not simply be their payoffs as an unintegrated seller.

6 Appendix

This appendix describes the numerical techniques used to determine the sellers’ premerger and postmerger equilibrium behavior.

I first describe the “truncated and modified” Beta distribution that is used in the numerical analysis. The Beta distribution has support \([0, 1]\), includes the Uniform distribution as a special case, and varies smoothly in its strictly positive parameters \(a\) and \(b\). By varying the parameters, the Beta distribution can take on a variety of shapes and can be made to look like many other distributions. Rather than working with the two Beta parameters \(a\) and \(b\), one can equivalently work with the mean and variance, which uniquely determine \(a\) and \(b\), and hence uniquely determine the distribution.

While the Beta distribution is quite flexible, its density has two features that cause difficulties for the numerical procedures used to compute the premerger equilibria. First, for some parameter values the density is zero at the lower support. Not only does this feature violate an assumption in the existence result in Lebrun [1999], it causes the inverse price-setting functions to have infinite slope at that point, which causes the approximate solutions to move to excessively high costs as the price increases. Even if the density is positive but extremely small at the lower support, this effect prevents the numerical procedure from ever finding a suitable approximate solution. I circumvent this problem by modifying the density by adding a small positive constant to it, with suitable scaling so that the area under the new density equals 1. Second, for some parameter values the density is infinite at the lower support. While this feature is inconsequential to the theory, it causes the inverse price-setting functions to have zero slope at that point, which prevents the approximate solutions from moving to higher costs as the price increases. I circumvent this problem by truncating the distribution so that the support is \([\gamma, 1-\gamma]\), where \(\gamma > 0\) is small.17

Specifically, the sellers’ costs are assumed to be drawn from distributions \(G(c)\) that are members of a family of distributions created by a linear combination of a Uniform distribution and a distribution \(F(c)\) that is a member of the family of Beta distributions, both truncated to \([\gamma, 1-\gamma]\), so that

\[
G(c) = \theta \left( \frac{c - \gamma}{1 - 2\gamma} \right) + (1 - \theta) \left( \frac{F(c) - F(\gamma)}{F(1-\gamma) - F(\gamma)} \right).
\]

\(\theta \in (0, 1)\) and \(\gamma\) are chosen such that the feasible set of means and variances of the family of distributions \(G(c)\) is close to the feasible set of means and variances of the family of standard Beta distributions. I use \(\theta = 0.01\) and \(\gamma = 10^{-13}\) in the computations.

In the premerger setting, I use fourth-order Runge-Kutta18 to approximate the solution to the system of differential equations that emerges from the sellers’ first-order conditions for profit maximization

\[
\begin{align*}
\varphi'_2(p) &= \frac{1 - F_2(\varphi_2(p))}{(p - \varphi_1(p)) F'_2(\varphi_2(p))} \\
\varphi'_1(p) &= \frac{1 - F_1(\varphi_1(p))}{(p - \varphi_2(p)) F'(\varphi_1(p))}
\end{align*}
\]

17While it seems that the truncation should handle both problems on its own, in practice too large of a truncation is necessary to avoid the first problem of the density’s being too small.

18See Carnahan, Luther, and Wilkes [1990] for an overview.
with boundary conditions \( \varphi_1(\pi) = \varphi_2(\pi) = \pi \) and \( \varphi_1(p^*) = \varphi_1(p^*) = \bar{c} \). Lebrun [1999] proves that \( \pi = \bar{c} \), so the one remaining unknown that affects the solution is the price set by sellers with the lowest possible cost, \( \bar{c} \).

One approach is to numerically solve the system starting from the known highest price. Previous authors have indicated that this approach is problematic because the formulas for the derivatives in the system (3) yield \( \varphi'_1(\pi) = 0 \). However, the values of the derivatives at that point can be determined using L'Hopital's rule, and the algorithm can be directed to use those values at that point. A more serious problem seems to be that once the algorithm is off the “true” solution the values of the derivatives become quite large and sometimes negative. This behavior destabilizes the numerical solution and prevents convergence.

To circumvent the system’s instability when moving “backward” from the known highest price, a second approach is to start from the unknown lowest price.\(^{19} \) The instability of the backward approach suggests that the “forward” approach will be quite stable. Moreover, while by design the forward numerical solution will reach the highest price \( \pi \) exactly, it will reach the highest cost \( \bar{c} \) inexacty. Hence the computer will never try to evaluate the derivatives in the system (3) and return \( \frac{\pi}{p} \). For these reasons I use the forward approach.

One downside of the forward approach is that one must search for the true value of the lowest price. To determine \( \bar{p} \), first note that for a given lowest price \( \bar{p} \), there exists a unique solution to the system of equations in (3). However, \( \bar{p}^* \) is the only value of \( \bar{p} \) for which the unique solution also satisfies the boundary condition \( \varphi_1(\pi) = \varphi_2(\pi) = \pi \). If \( \bar{p} \) is chosen below \( \bar{p}^* \), then the solution will violate at least one of three conditions: the price-setting function will not be monotonic, it will at some points specify a price less than the seller’s cost, or it will specify a price greater than \( \pi \). If \( \bar{p} \) is chosen above \( \bar{p}^* \), then the solution will not violate the three preceding conditions, but it will fail to satisfy the boundary condition \( \varphi_1(\pi) = \varphi_2(\pi) = \pi \).

The difference in behavior of the solution on opposite sides of \( \bar{p}^* \) provides a simple means of converging to the correct solution. For each configuration of the sellers’ cost distributions, I start with an initial guess of the lowest price equal to the midpoint of the distributions’ common support. I set the algorithm so that it takes 400 steps between the lowest and highest prices, and so that the solution converges if both costs at the highest price are within 0.005 of \( \pi \). If the solution does not converge after 40 bisections, then the number of steps is increased by 400, the algorithm repeats, and so on. Every solution converged with 1600 steps or fewer.

After determining the equilibrium price-setting functions for a particular combination of the two sellers’ cost distributions, the expected price and the sellers’ expected profits and market shares were determined through Monte Carlo methods. Specifically, 1000 pairs of pseudorandom draws were taken from \( U \{0, 1\} \) to represent realizations from each seller’s cumulative distribution function. A root-finding procedure using the bisection method was then used to generate the cost realizations associated with each pair of cdf draws. Each pair of cost draws was then associated with a pair of prices using the computed equilibrium price-setting functions. The seller offering the lowest price was the winner, and measures of its profit and market share and of the market price were appropriately incremented. The same set of cdf draws was used for each combination of the sellers’ cost distributions.

In the postmerger setting, I use a root-finding procedure to solve for the price that satisfies the unintegrated seller’s first-order condition for profit maximization when it has cost \( c_U \),

\[
1 - F_U(p) - (p - c_U)F_U'(p) = 0. \tag{4}
\]

\(^{19}\)An observed difficulty with this approach occurs if \( F_U'(c) = 0 \), in which case the system (3) has a singularity at \( c \). However, the existence result in Lebrun [1999] specifies that \( F_U'(c) > 0 \).
I solve equation (4) using the bisection method with initial points $c_U$ and $\bar{c}$, which examination shows bracket the root. The root-finding procedure determines a price for any particular cost. I estimate the expected values from the average of the values found by taking 1000 pairs of cost draws from the distributions $F_I$ and $F_U$. These cost draws are the same draws that were used to determine the premerger price-setting behavior, in order to accurately compare the premerger and postmerger behavior.

References


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20 Visual inspection revealed that the profit function is quasiconcave for $c_U = \bar{c}$, so the price that is optimal for $c_U = \bar{c}$ is unique. That the second-order condition holds for each distribution for every $c_U$ was found by visual inspection of the second-order condition for every price between the price that is optimal for $c_U = \bar{c}$ and the price $\bar{c}$ that is optimal for $c_U = \bar{c}$. It is appropriate to look only at this set of prices, because for all costs higher than $\bar{c}$ any proposed maxima must occur at higher prices. Visual inspection revealed that $[1 - F_I(p)] F'_I(p) + 2 F'_J(p)^2 \geq 0$ for all prices in the relevant range, which is the condition for which the second-order condition holds anywhere the first-order condition holds, for some cost.


Figure 1. Feasible Means and Variances: Standard Beta distribution and "Truncated and Modified" Beta distribution.

Figure 2. Change in Merged Firm's Joint Profits as a Function of Integrated Seller's Premerger Market Share
Figure 3. Profitability of Merger with Seller 1, as a Function of Differences in Means and Variances of Sellers' Costs

Figure 4. Buyer's Preferred Merger Partner, as a Function of Differences in Means and Variances of Sellers' Costs
Figure 5. Change in Total Welfare as a Function of the Integrated Seller’s Premerger Market Share
Table 1
Curve-Fitting Results Using Means and Variances of Sellers’ Costs

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<th>Buyer’s Premerger Expected Profit $\pi_\beta$</th>
<th>Seller 1’s and Buyer’s Premerger Integrated Expected Profit $\pi_1 + \pi_\beta$</th>
<th>Seller 1’s and Buyer’s Postmerger Integrated Expected Profit $\pi'_1$</th>
<th>Seller 2’s Postmerger Unintegrated Expected Profit $\pi'_2$</th>
<th>Profitability of Merger Between Seller 1 and Buyer $\pi'<em>1 - (\pi_1 + \pi</em>\beta)$</th>
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<td>Mean,*Variance, $\pi_1$</td>
<td>0.28288243</td>
<td>-0.22341980</td>
<td>0.05946263</td>
<td>-0.24575642</td>
<td>0.02926897</td>
<td>-0.30521907</td>
</tr>
<tr>
<td>Mean,*Mean, $\pi_1$</td>
<td>-0.82124373</td>
<td>0.80047393</td>
<td>-0.02076980</td>
<td>-0.09451221</td>
<td>-0.82021708</td>
<td>-0.07374241</td>
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<tr>
<td>Mean,*Variance, $\pi_2$</td>
<td>-0.15268603</td>
<td>0.57136306</td>
<td>0.41867703</td>
<td>-0.05741171</td>
<td>-0.34251421</td>
<td>-0.47608874</td>
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<tr>
<td>Variance, $\pi_2$</td>
<td>-0.26182039</td>
<td>-1.12659301</td>
<td>-1.38841339</td>
<td>-0.61649710</td>
<td>0.02908154</td>
<td>0.77191629</td>
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<tr>
<td>Variance,*Mean, $\pi_2$</td>
<td>-0.62562474</td>
<td>0.57427703</td>
<td>-0.05134771</td>
<td>0.42815657</td>
<td>0.03343110</td>
<td>0.47950427</td>
</tr>
<tr>
<td>Variance,*Variance, $\pi_2$</td>
<td>-2.47421987</td>
<td>4.43381588</td>
<td>1.9595601</td>
<td>-0.38526650</td>
<td>-0.90839800</td>
<td>-2.3448251</td>
</tr>
<tr>
<td>Mean, $\pi'_2$</td>
<td>0.32394395</td>
<td>-0.26670622</td>
<td>0.05723773</td>
<td>0.07193265</td>
<td>0.53369584</td>
<td>0.01469492</td>
</tr>
<tr>
<td>Mean,*Variance, $\pi_2$</td>
<td>0.07348458</td>
<td>-0.22696522</td>
<td>-0.15348064</td>
<td>0.01957910</td>
<td>0.42400200</td>
<td>0.17305974</td>
</tr>
<tr>
<td>Variance, $\pi'_2$</td>
<td>1.37918742</td>
<td>-1.13854754</td>
<td>0.24063988</td>
<td>0.01904060</td>
<td>2.52801022</td>
<td>-0.22159929</td>
</tr>
<tr>
<td>Data Points</td>
<td>10,201</td>
<td>10,201</td>
<td>10,201</td>
<td>10,201</td>
<td>10,201</td>
<td>10,201</td>
</tr>
<tr>
<td>Adjusted R$^2$</td>
<td>0.988</td>
<td>0.988</td>
<td>0.996</td>
<td>0.998</td>
<td>0.983</td>
<td>0.955</td>
</tr>
</tbody>
</table>

Table 2
Percentage of Feasible $(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)$-tuples with Strictly Positive Derivatives

<table>
<thead>
<tr>
<th></th>
<th>Seller 1’s Premerger Expected Profit $\pi_1$</th>
<th>Buyer’s Premerger Expected Profit $\pi_\beta$</th>
<th>Seller 1’s and Buyer’s Premerger Integrated Expected Profit $\pi_1 + \pi_\beta$</th>
<th>Seller 1’s and Buyer’s Postmerger Integrated Expected Profit $\pi'_1$</th>
<th>Seller 2’s Postmerger Unintegrated Expected Profit $\pi'_2$</th>
<th>Profitability of Merger Between Seller 1 and Buyer $\pi'<em>1 - (\pi_1 + \pi</em>\beta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d/d\mu_1$</td>
<td>3.73%</td>
<td>1.27%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>5.30%</td>
<td>1.66%</td>
</tr>
<tr>
<td>$d/d\sigma_1^2$</td>
<td>100.00%</td>
<td>6.27%</td>
<td>76.91%</td>
<td>3.46%</td>
<td>100.00%</td>
<td>2.67%</td>
</tr>
<tr>
<td>$d/d\mu_2$</td>
<td>93.96%</td>
<td>1.27%</td>
<td>0.00%</td>
<td>1.66%</td>
<td>90.79%</td>
<td>99.34%</td>
</tr>
<tr>
<td>$d/d\sigma_2^2$</td>
<td>81.03%</td>
<td>6.18%</td>
<td>0.57%</td>
<td>100.00%</td>
<td>93.70%</td>
<td>99.78%</td>
</tr>
</tbody>
</table>